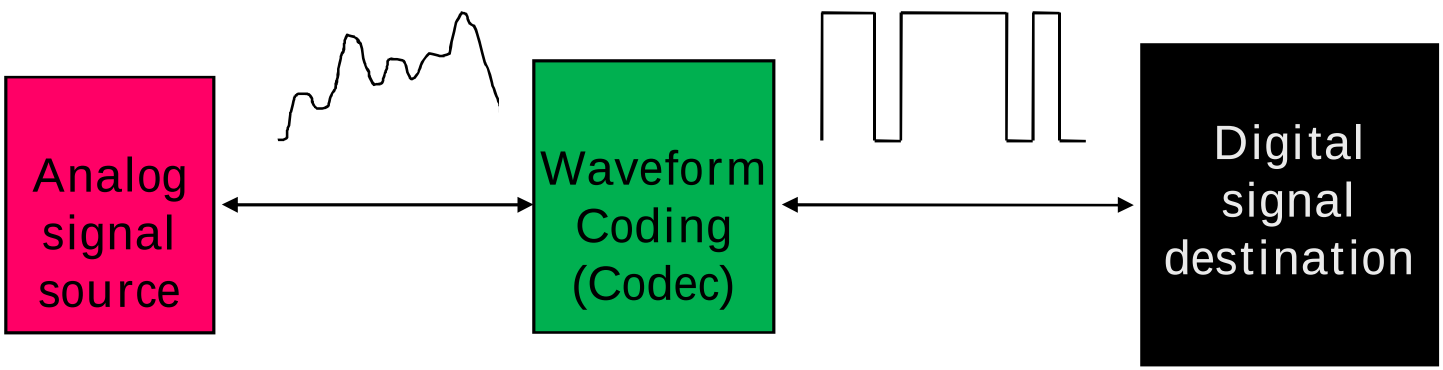
UNIT 1: Sampling and Quantization

Introduction

- Digital representation of analog signals

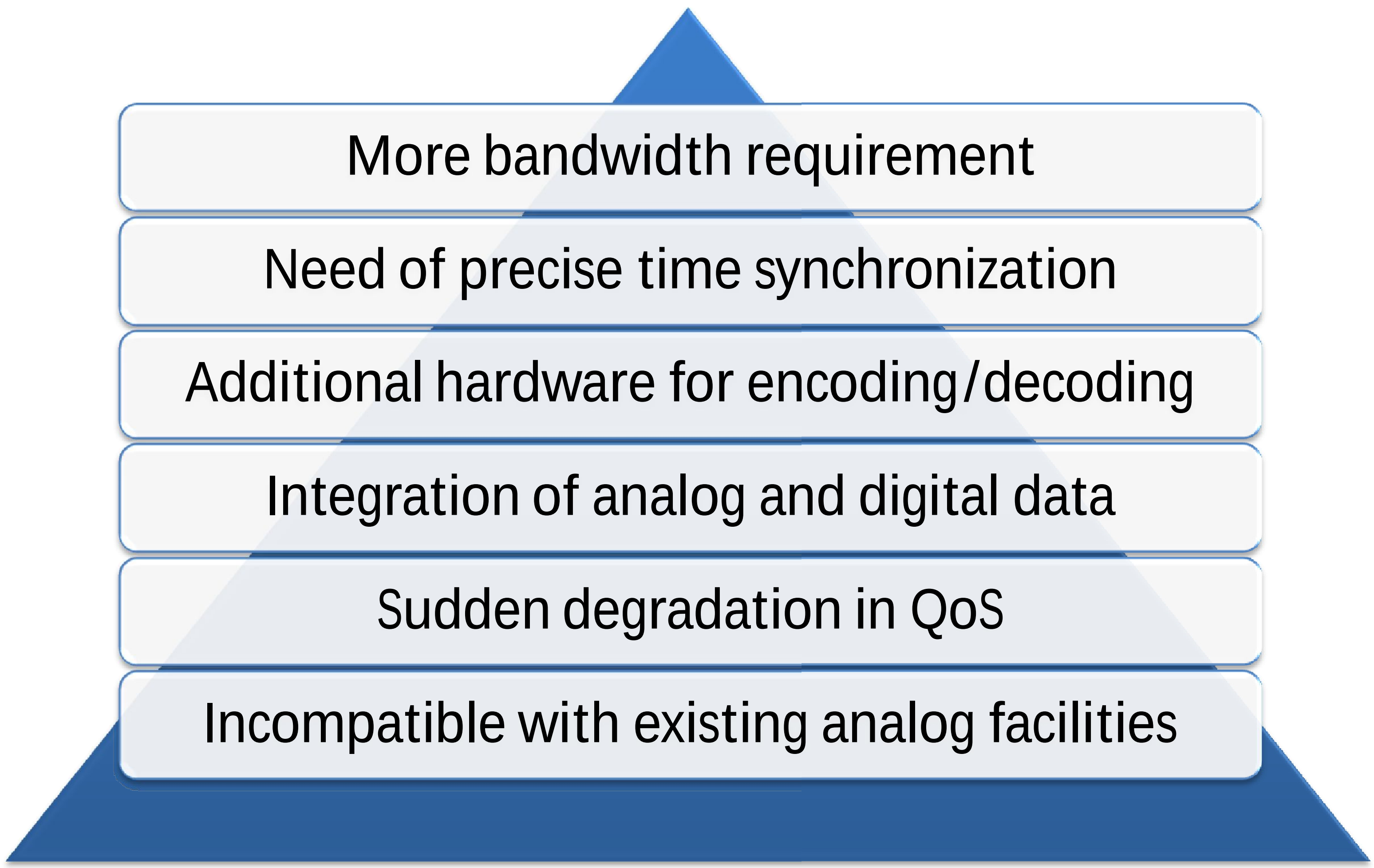


Analog-to-Digital Encoding

Advantages of Digital Transmissions

- Noise immunity
- Error detection and correction
- Ease of multiplexing
- Integration of analog and digital data
- Use of signal regenerators
- Data integrity and security
- Ease of evaluation and measurements
- More suitable for processing ....

Disadvantages of Digital Transmissions



A Typical Digital Communication Link

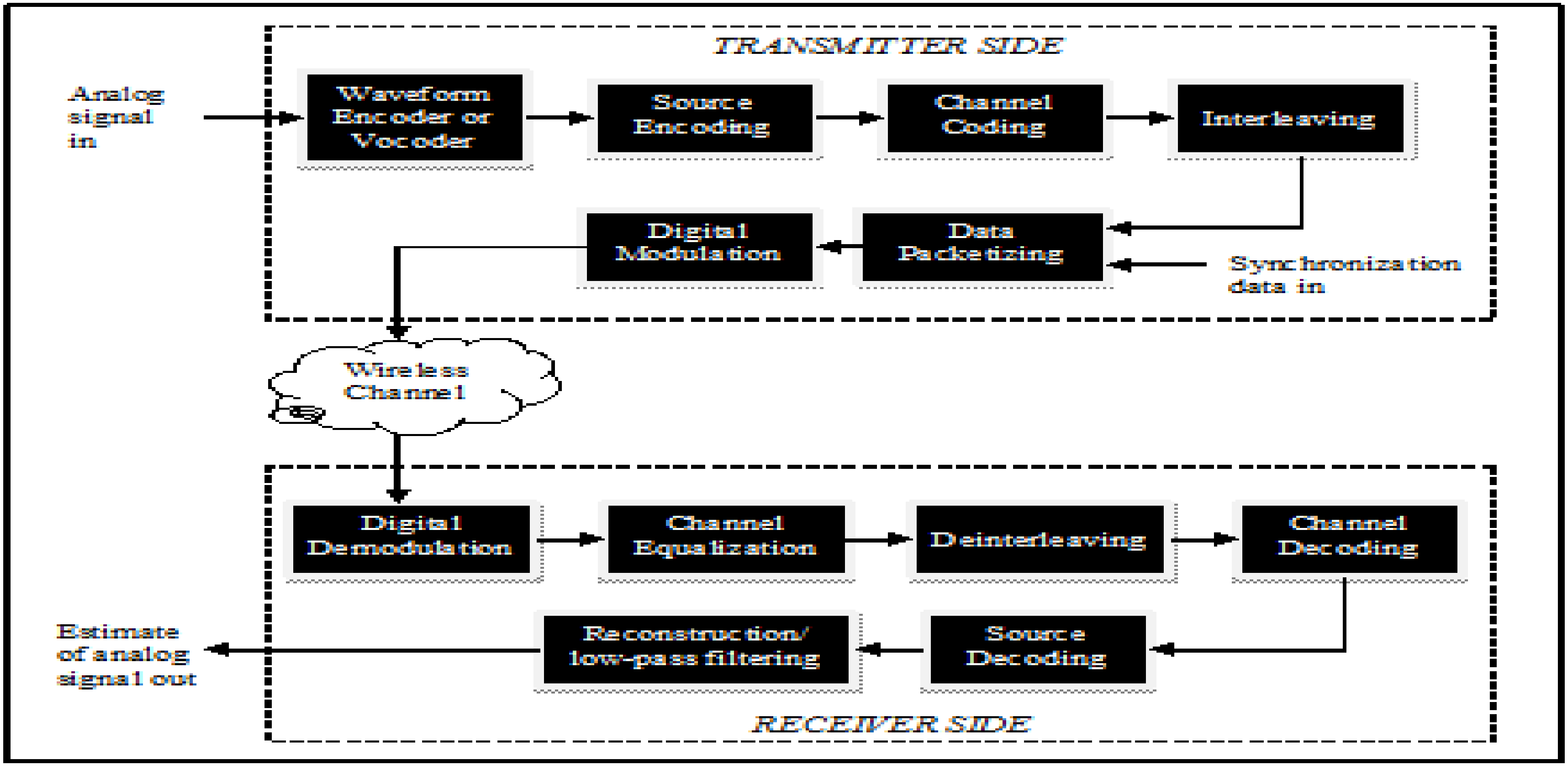


Fig. 2 Block Diagram

Formatting

aracter coding
mpling
antization
se code modulation
PCM)

Source Coding

Predictive coding
Block coding
Variable length coding
Synthesis/analysis coding
Lossless compression
Lossy compression

Baseband Signaling

PCM waveforms (line codes)
- Nonreturn-to-zero (NRZ)
- Return-to-zero (RZ)
- Phase encoded
- Multilevel binary

M-ary pulse modulation
- PAM, PPM, PDM

Equalization

Maximum-likelihood sequence estimation (MLSE)

Equalization with filters
- Transversal or decision feedback
- Preset or Adaptive
- Symbol spaced or fractionally spaced

Bandpass Signaling

Coherent

e shift keying (PSK)
uency shift keying (FSK)
litude shift keying (ASK)
inuous phase modulation (CPM)
ids

Noncoherent

Differential phase shift keying (DPSK)
Frequency shift keying (FSK)
Amplitude shift keying (ASK)
Continuous phase modulation (CPM)
Hybrids

Channel Coding

Waveform

M-ary signaling
Antipodal
Orthogonal
Trellis-coded modulation

Structured Sequences

Block
Convolution
Turbo

Synchronization

Frequency synchronization
Phase synchronization
Symbol synchronization
Frame synchronization
Network synchronization

Multiplexing/Multiple Access

Frequency division (FDM/FDMA)
Time division (TDM/TDMA)
Code division (CDM/CDMA)
Space division (SDMA)
Polarization division (PDMA)

Spreading

Direct sequencing (DS)
Frequency hopping (FH)
Time hopping (TH)
Hybrids

Encryption

Block
Data stream

Figure 1.3 Basic digital communication transformations.

Basic Digital Communication Transformations

- Formatting/Source Coding
- Transforms source info into digital symbols (digitization)
- Selects compatible waveforms (matching function)
- Introduces redundancy which facilitates accurate decoding despite errors

It is essential for reliable communication

- Modulation/Demodulation
- Modulation is the process of modifying the info signal to facilitate transmission
- Demodulation reverses the process of modulation. It involves the detection and retrie of the info signal
  - Types
  - Coherent: Requires a reference info for detection
  - Noncoherent: Does not require reference phase information

Basic Digital Communication Transformations

– Coding/Decoding

Translating info bits to transmitter data symbols

Techniques used to enhance info signal so that they are less vulnerable to channel impairment (e.g. noise, fading, jamming, interference)

- Two Categories
  - *Waveform Coding*
- Produces new waveforms with better performance
  - *Structured Sequences*

Involves the use of redundant bits to determine the occurrence of error (an sometimes correct it)

– Multiplexing/Multiple Access Is synonymous with resource sharing with other users

– Frequency Division Multiplexing/Multiple Access (FDM/FDMA

Practical Aspects of Sampling

1. Sampling Theorem

2 .Methods of Sampling

3. Significance of Sampling Rate

4. Anti-aliasing Filter

5 . Applications of Sampling Theorem – PAM/TDM

Sampling

Sampling is the processes of converting continuous-time analog signal, $x_a(t)$, into a discrete-time s
by taking the “samples” at discrete-time intervals

- Sampling analog signals makes them discrete in time but still continuous valued
- If done properly (***Nyquist theorem*** is satisfied), sampling does not introduce distortion

Sampled values:

- The value of the function at the sampling points

Sampling interval:

- The time that separates sampling points (interval b/w samples), T_s
- If the signal is slowly varying, then fewer samples per second will be required than if the wavef
  is rapidly varying
- So, the optimum sampling rate depends on the *maximum frequency* component present in the
  signal

Analog-to-digital conversion is (basically) a 2 step process:

- Sampling
  - Convert from continuous-time analog signal $x_a(t)$ to discrete-time continuous value signal x
- Is obtained by taking the “samples” of $x_a(t)$ at discrete-time intervals, T_s

Quantization

- Convert from discrete-time continuous valued signal to discrete time discrete valued signal

Sampling

ling Rate (or sampling frequency f_s):

ne rate at which the signal is sampled, expressed as the number of samples per second eciprocal of the sampling interval), ***$1/T_s = f_s$***

st Sampling Theorem (or Nyquist Criterion):

the sampling is performed at a proper rate, no info is lost about the original signal and it can be operly reconstructed later on

atement:

"If a signal is sampled at a rate at least, but not exactly equal to twice the max frequency onent of the waveform, then the waveform can be exactly reconstructed from the samples without any distortion"

$$f_s \geq 2 f_{\max}$$

..... Sampling Theorem

Sampling Theorem for Bandpass Signal - If an analog information signal containing no frequency outside the specified bandwidth W Hz, it may be reconstructed from its samples at a sequence of points spaced 1/(2W) seconds apart with zero-mean squared error.

The minimum sampling rate of (2W) samples per second, for an analog signal bandwidth of W Hz, is called the **Nyquist rate**.

The reciprocal of Nyquist rate, 1/(2W), is called the **Nyquist interval**, that is, $T_s = 1/(2W)$.

The phenomenon of the presence of high-frequency component in the spectrum of the original analog signal is called aliasing or simply foldover.

Sampling Theorem

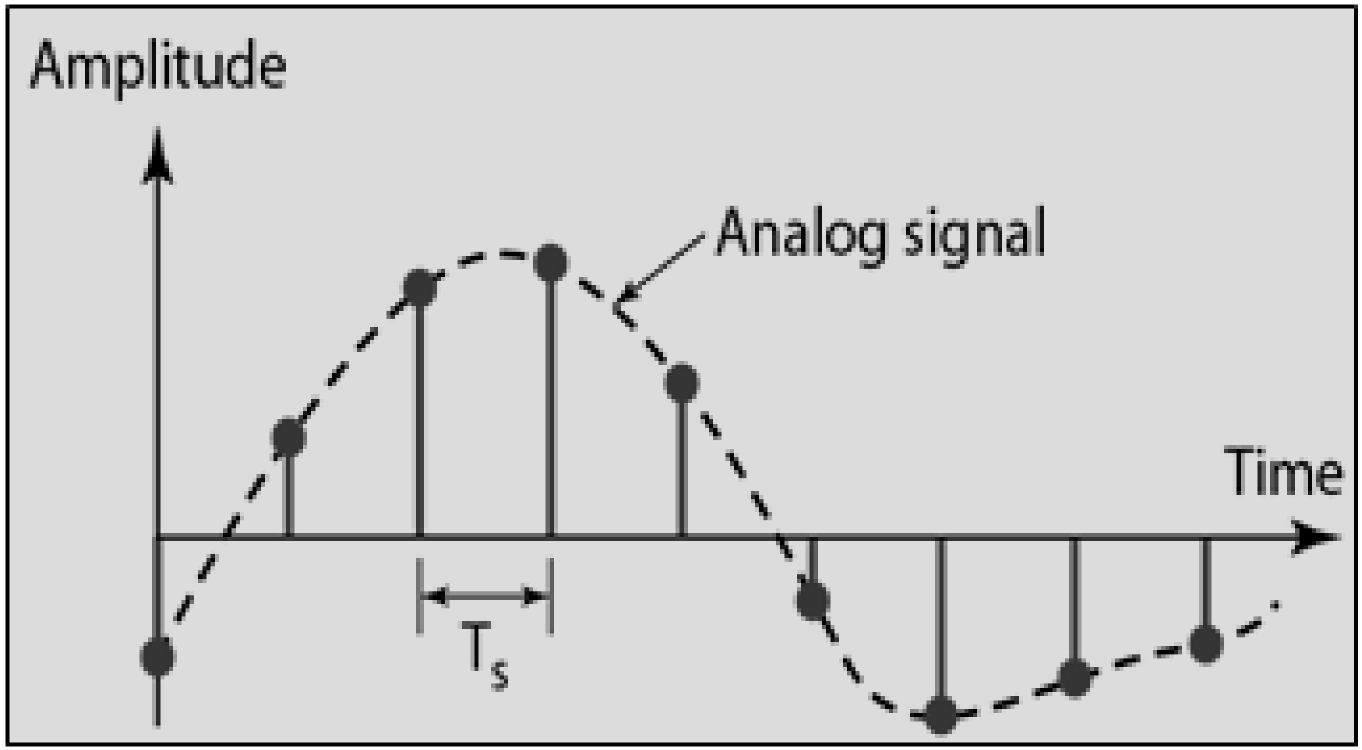
Sampling Theorem for Baseband Signal - A baseband signal having no frequency components higher than f_m Hz may be completely recovered from the knowledge of its samples taken at a rate of at least $2 f_m$ samples per second, that is, sampling frequency $\boldsymbol{f_s \geq 2 f_m}$.

The minimum sampling rate $f_s = 2 f_m$ samples per second is called the Nyquist sampling rate.

A baseband signal having no frequency components higher than f_m Hz is completely described by its sample values at uniform intervals less than or equal to $1/(2f_m)$ seconds apart, that is, the sampling interval $T_s \leq 1/(2f_m)$ seconds.

Methods of Sampling

Ideal sampling - an impulse at each sampling instant

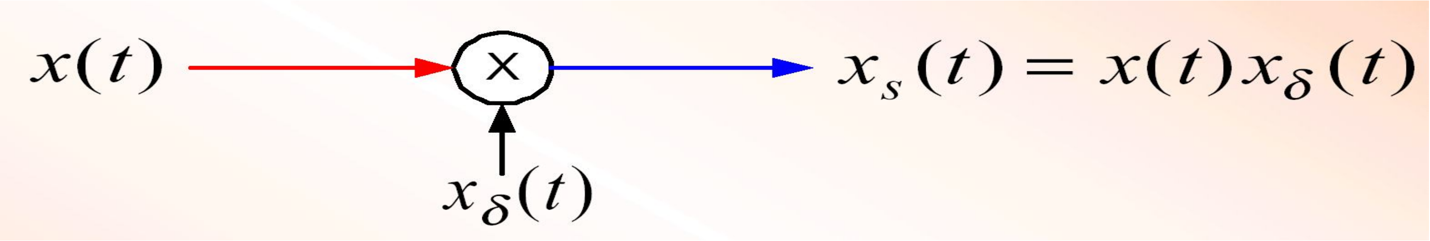


Ideal Sampling

Ideal Sampling (or Impulse Sampling)

Is accomplished by the multiplication of the signal *x(t)* by the uniform train of impulses (comb function)

Consider the instantaneous sampling of the analog signal *x(t)*



Train of impulse functions select sample values at regular intervals

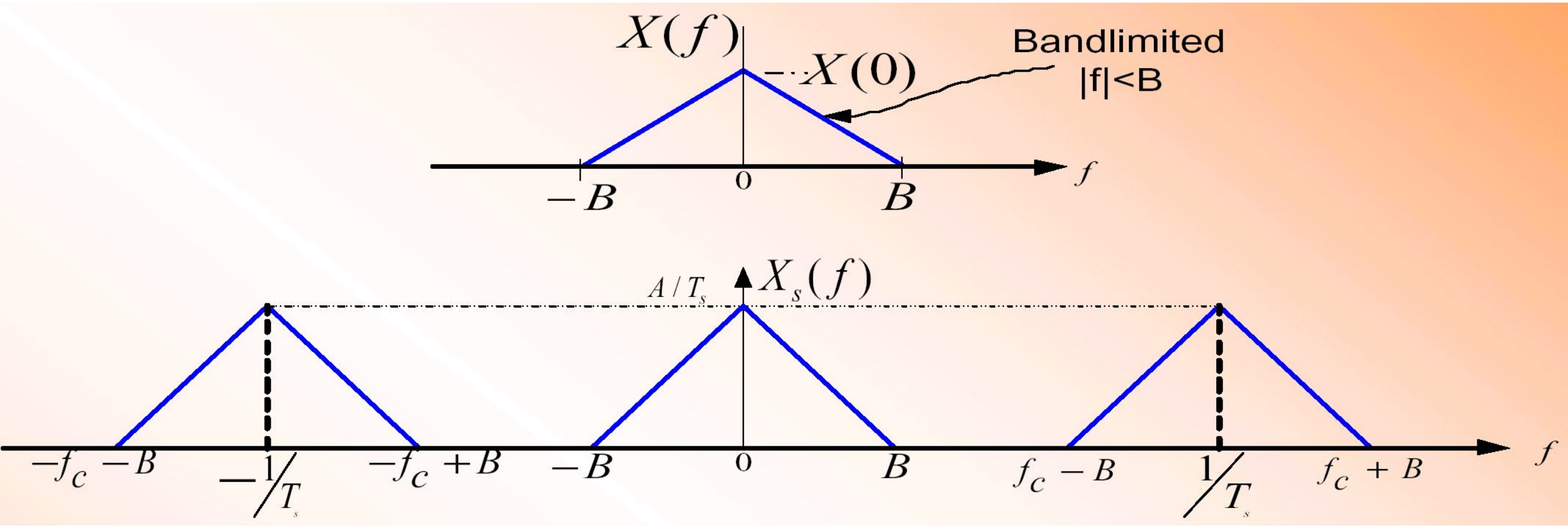
$$x_s(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$$

Fourier Series representation:

$$\sum_{n=-\infty}^{\infty}\delta(t-nT_s) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}e^{jn\omega_s t}, \qquad \omega_s = \frac{2\pi}{T_s}$$

Ideal Sampling (or Impulse Sampling)

is shows that the Fourier Transform of the sampled signal is the Fourier Transform of the original
nal at rate of $1/T_s$



Ideal Sampling (or Impulse Sampling)

s long as $f_s > 2f_m$, no overlap of repeated replicas $X(f - n/T_s)$ will occur in $X_s(f)$

Minimum Sampling Condition:

$$f_s - f_m > f_m \Rightarrow f_s > 2f_m$$

Sampling Theorem: A finite energy function *x(t)* can be completely ***reconstructed*** from ampled value *x(nTs)* with

$$(t) = \sum_{n=-\infty}^{\infty} T_s x(nT_s) \left\{ \frac{\sin\left[\frac{2\pi f(t - nT_s)}{2T_s}\right]}{\pi(t - nT_s)} \right\}$$

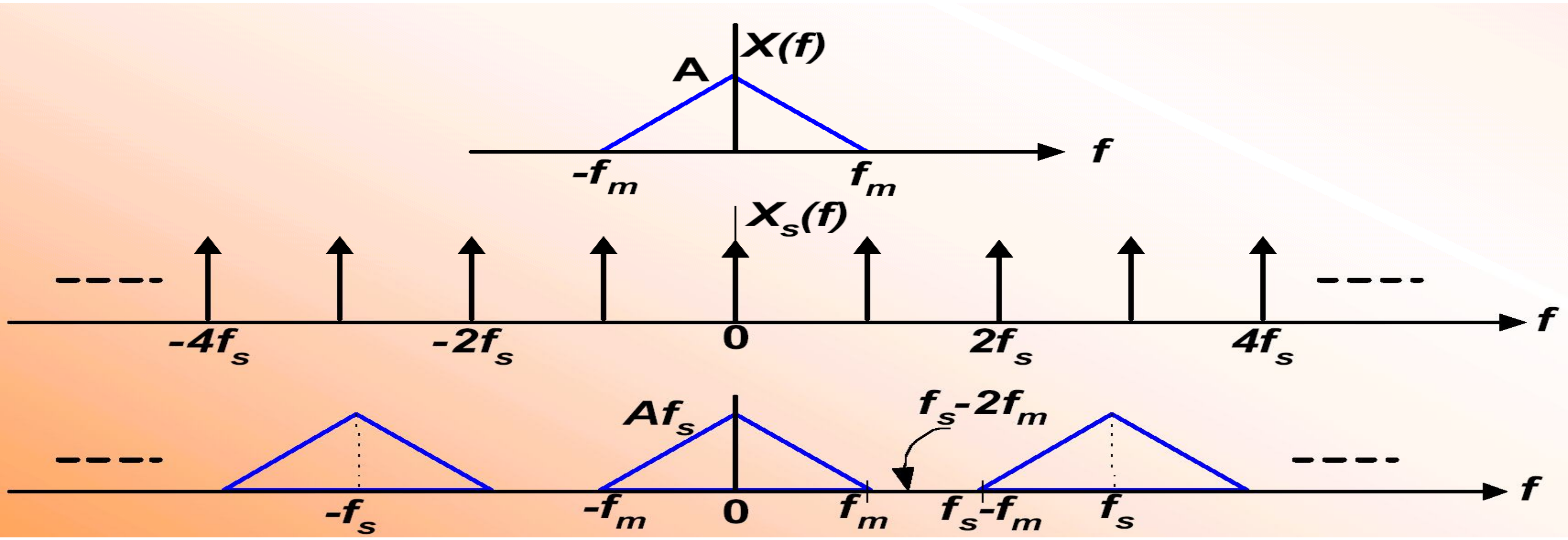
$$= \sum_{n=-\infty}^{\infty} T_s \, x(nT_s) \, \sin c(2 f_s(t - nT_s))$$

provided that =>

$$\frac{1}{f_s} = T_s \le \frac{1}{2 f_m}$$

Ideal Sampling (or Impulse Sampling)

is means that the output is simply the replication of the original signal at discrete intervals, e.g



T_s is called the ***Nyquist interval:*** It is the longest time interval that can be used for sampling a bar
signal and still allow reconstruction of the signal at the receiver without distortion

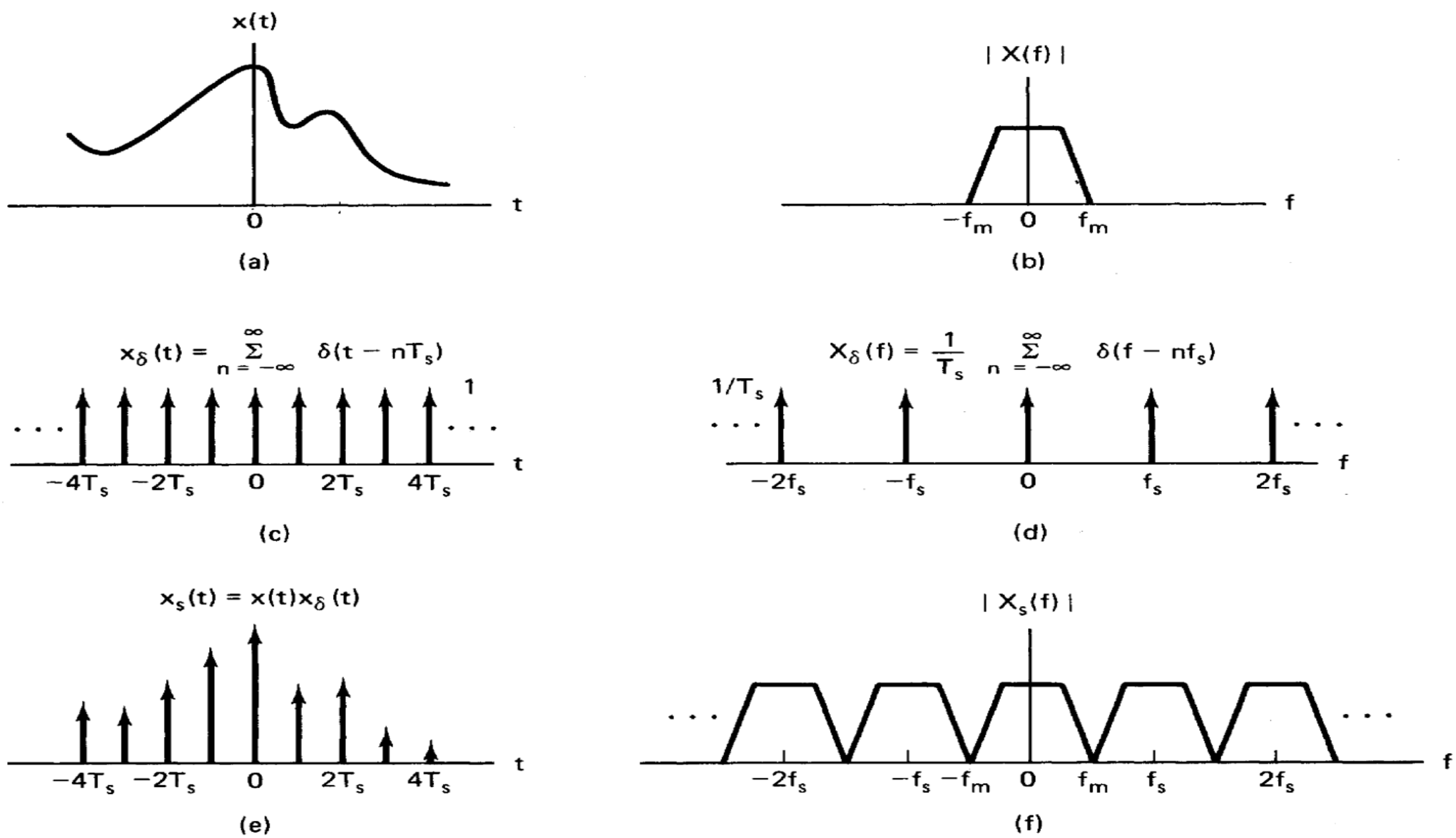
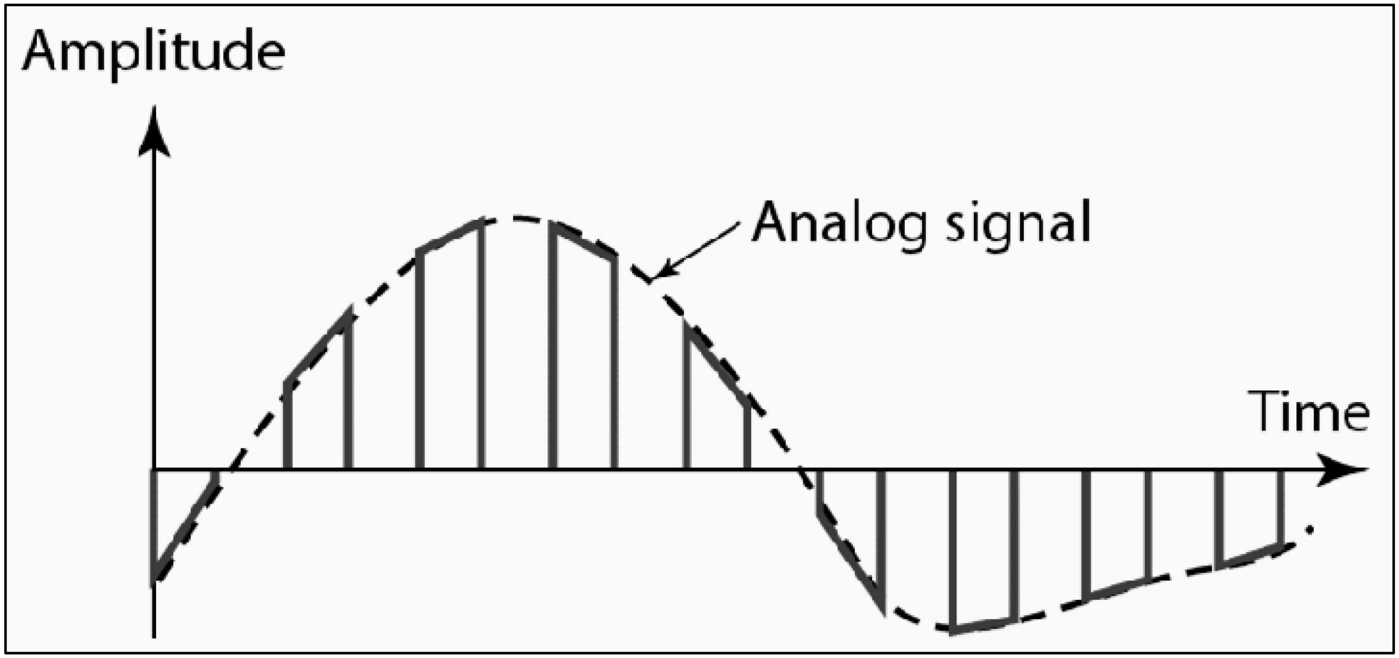


Figure 2.6 Sampling theorem using the frequency convolution property of the Fourier transform.

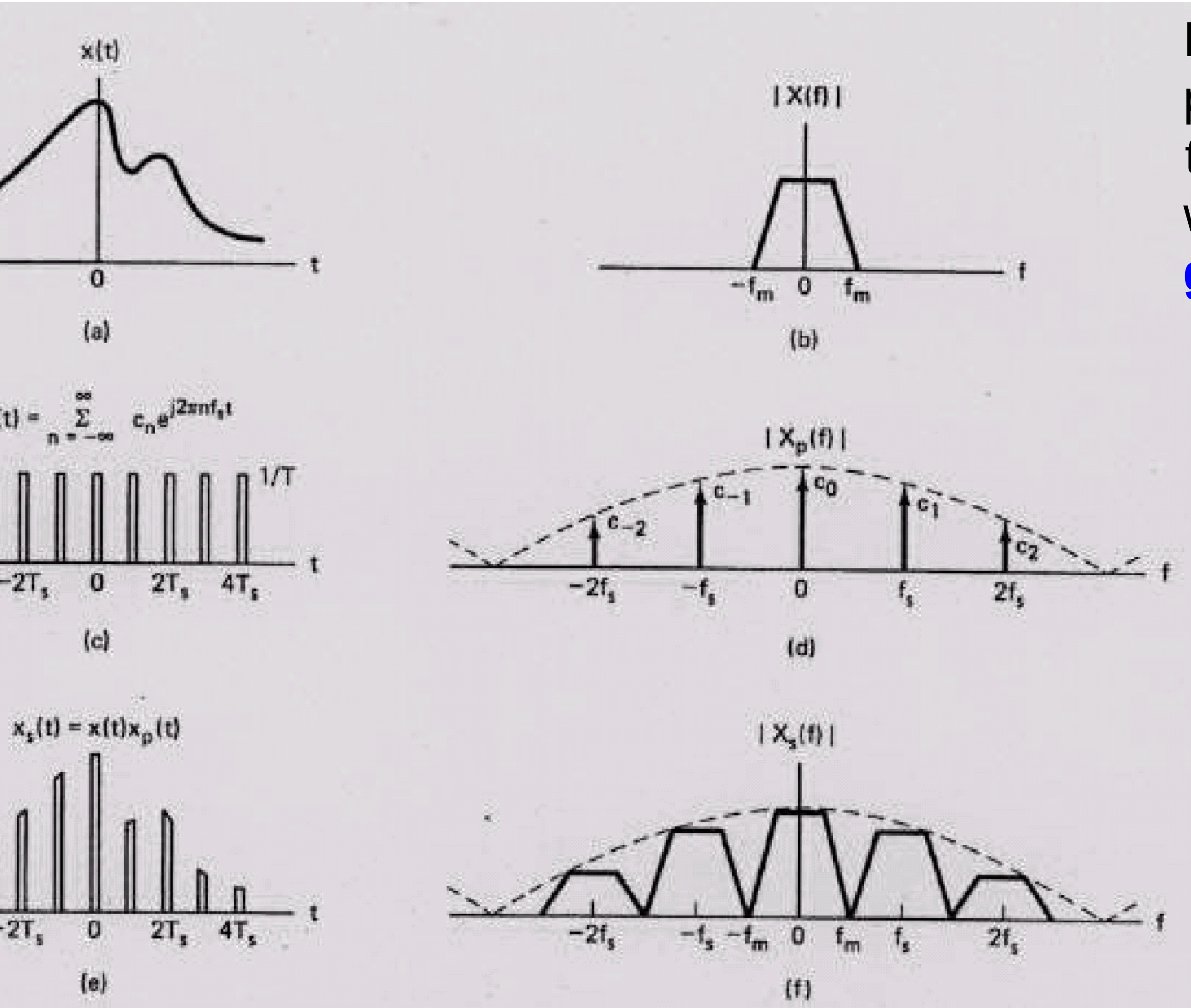
..... Methods of Sampling

Natural sampling - a pulse of short width with varying amplitude with natural tops



Natural Sampling

Natural Sampling



If we multiply $x(t)$ by a train of rectang pulses $x_p(t)$, we obtain a gated wavefo that approximates the ideal sampled waveform, known as **natural samplin gating (s**ee Figure 2.8)

$$x_s(t) = x(t)\,x_p(t)$$

$$= x(t)\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s}$$

$$X_s(f) = \Im[x(t)\,x_p(t)]$$

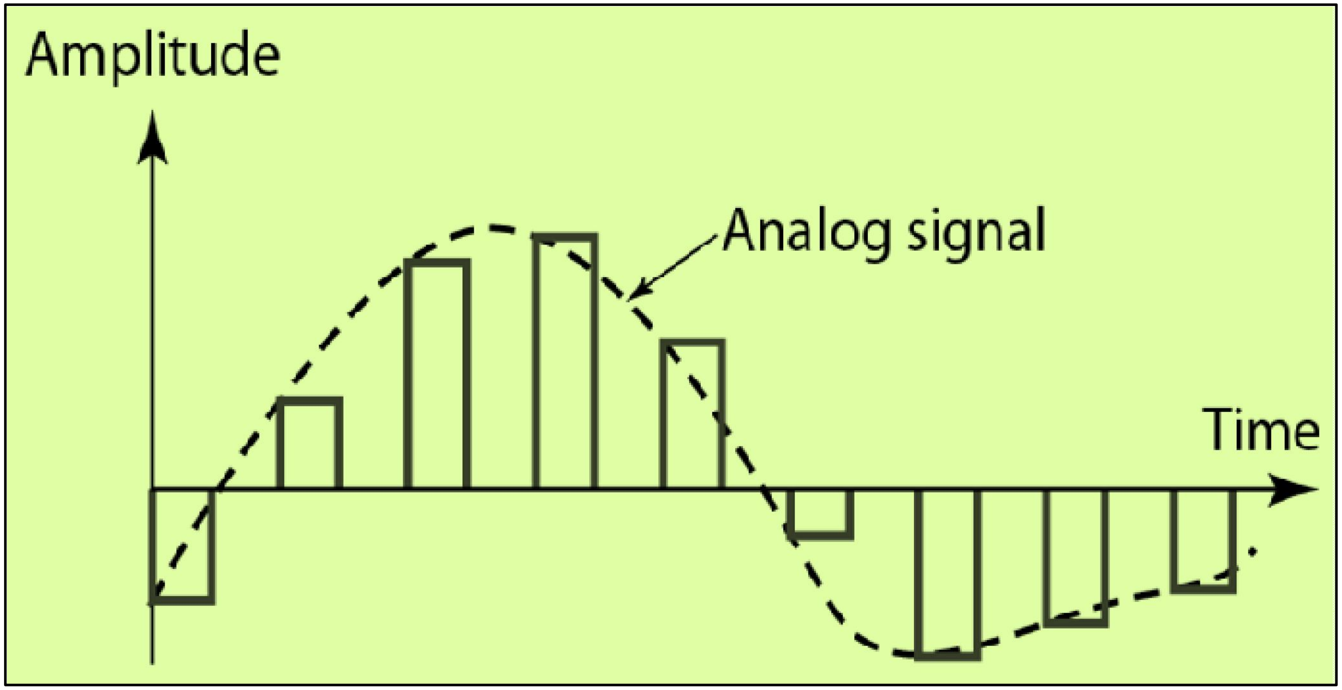
$$= \sum_{n=-\infty}^{\infty} c_n \Im[x(t)\,e^{j2\pi}$$

$$= \sum_{n=-\infty}^{\infty} c_n X[f - nf_s$$

- Each pulse in $x_p(t)$ has width T_s and amplitude $1/T_s$
- The top of each pulse follows the variation of the signal being sampled
- $X_s(f)$ is the replication of $X(f)$ periodically every f_s Hz
- $X_s(f)$ is weighted by $C_n \leftarrow$ Fourier Series Coeffiecient
- The problem with a natural sampled waveform is that the tops of the sample pulses are not flat
- It is not compatible with a digital system since the amplitude of each sample has infinite number of possible values
- Another technique known as ***flat top sampling*** is used to alleviate this problem

….. Methods of Sampling

Flat-top sampling - a pulse of short width with varying amplitude with flat tops



Flat-top Sampling

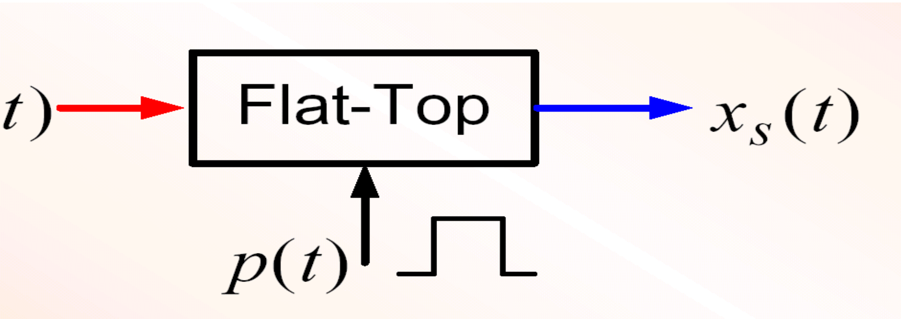
Flat-Top Sampling

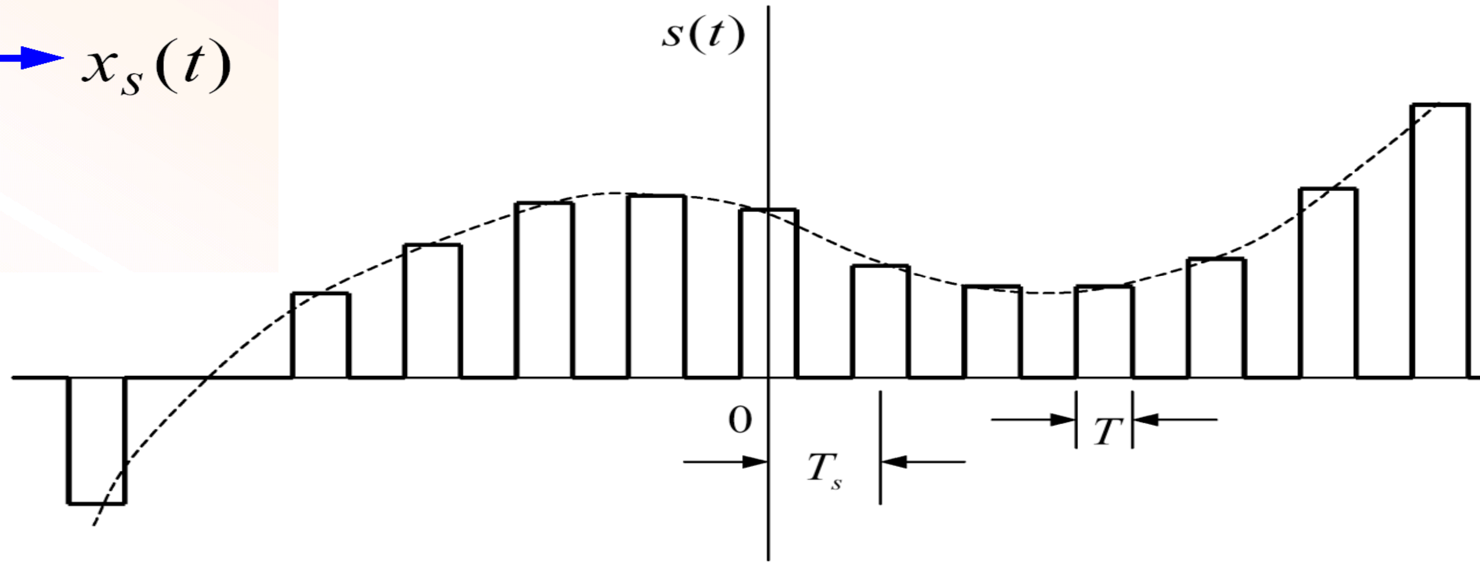
ere, the pulse is held to a constant height for the whole sample period

at top sampling is obtained by the convolution of the signal obtained after ideal mpling with a unity amplitude rectangular pulse, *p(t)*

is technique is used to realize ***Sample-and-Hold*** (S/H) operation

S/H, input signal is continuously sampled and then the value is held for as long as kes to for the A/D to acquire its value





Flat top sampling (Time Domain)

$$x'(t) = x(t)\delta(t)$$

$$x_s(t) = x'(t) * p(t)$$

$$= p(t) * x(t)\delta(t) = p(t) * \left[x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right]$$

Taking the Fourier Transform will result to

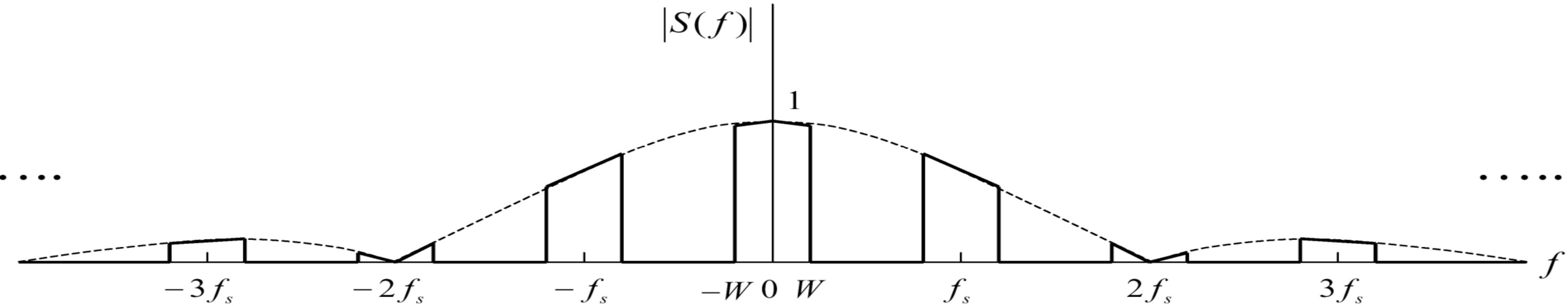
$$X_s(f) = \Im[x_s(t)]$$

$$= P(f)\,\Im\left[x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)\right]$$

$$= P(f)\,\Im\left[X(f) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f - nf_s)\right]$$

$$= P(f)\frac{1}{T_s}\sum_{n=-\infty}^{\infty}X(f - nf_s)$$

where *P(f)* is a *sinc* function

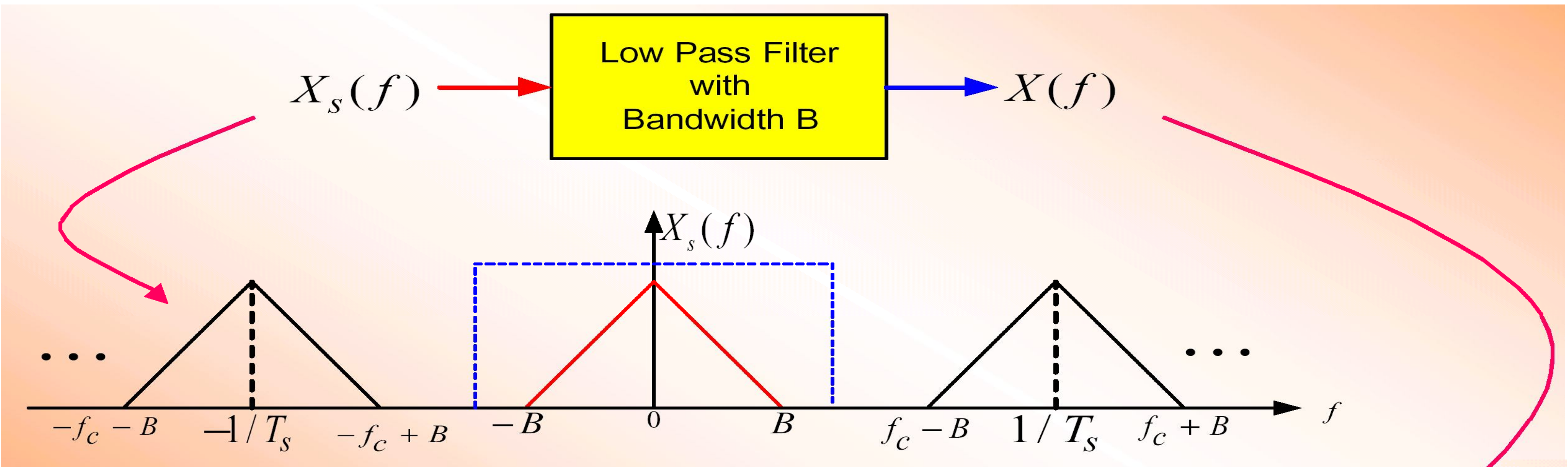


Flat top sampling (Frequency Domain)

- Flat top sampling becomes identical to ideal sampling as the width of the pulses become shorter

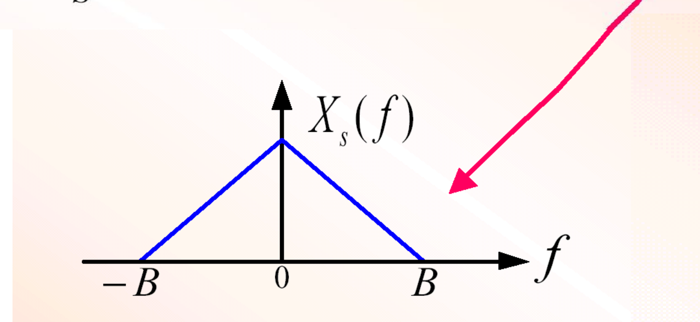
Recovering the Analog Signal

One way of recovering the original signal from sampled signal $X_s(f)$ is to pass it through a Low Pass Filter (LPF) as shown below



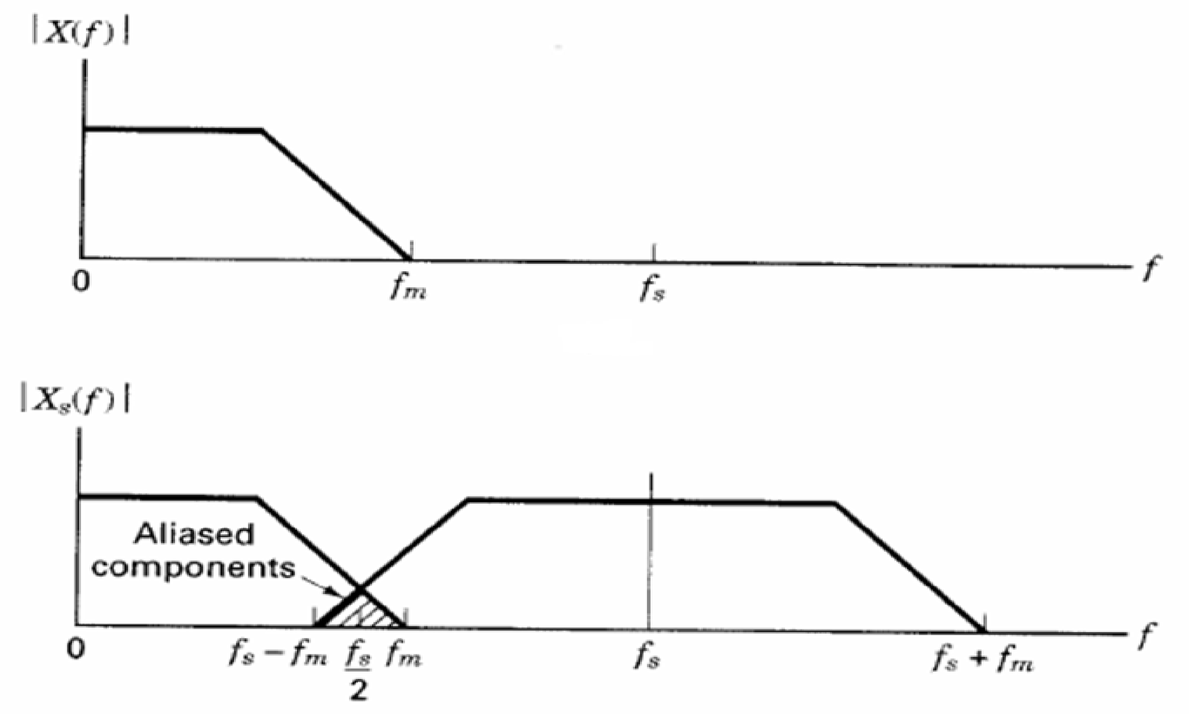
If $f_s > 2B$ then we recover *x(t)* ***exactly***

Else we run into some problems and signal is not fully recovered



Significance of Sampling Rate

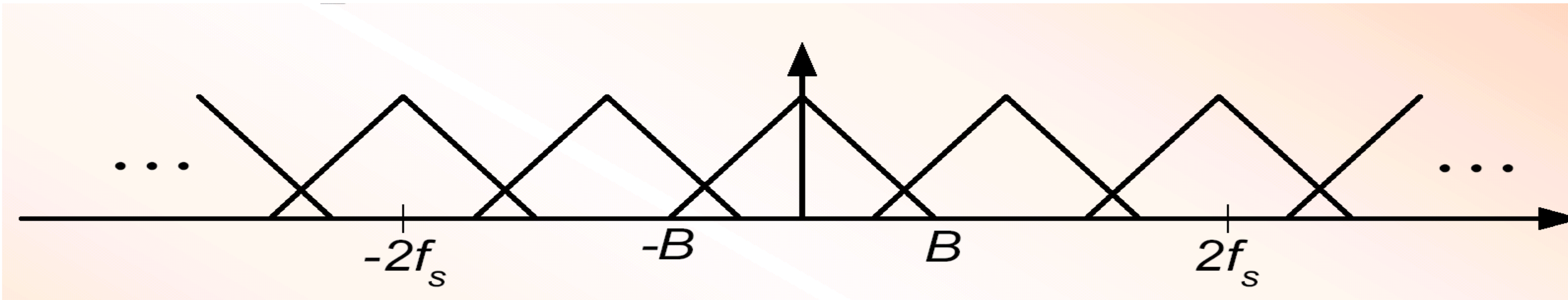
When $f_s < 2f_m$, spectral components of adjacent samples will overlap, known as aliasing



An Illustration of Aliasing

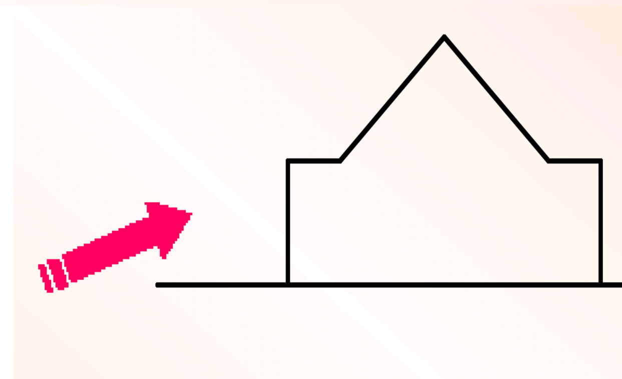
Undersampling and Aliasing

– If the waveform is ***undersampled*** (i.e. *fs* < *2B*) then there will be ***spectral overlap*** in the sample signal



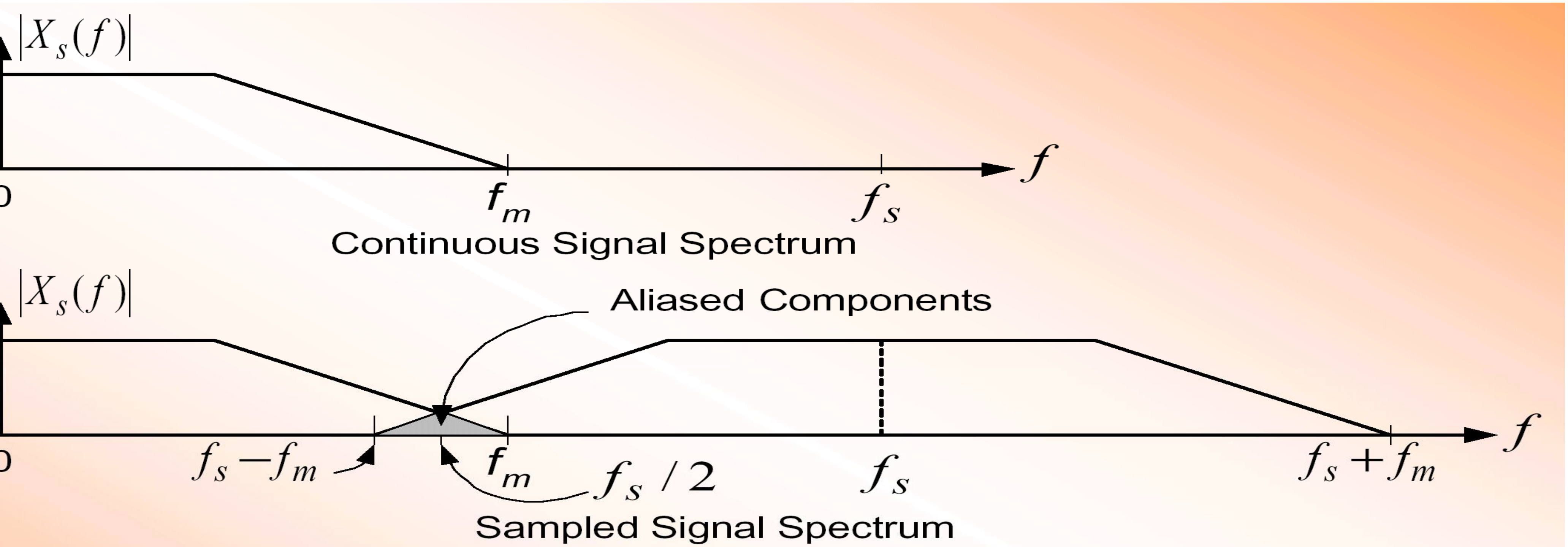
e signal at the output of the filter will be

different from the original signal spectrum



This is the outcome of ***aliasing***!

is implies that whenever the sampling condition is not met, an irreversible overlap of the spectral licas is produced



This could be due to:

1. *x(t)* containing higher frequency than were expected
2. An error in calculating the sampling rate

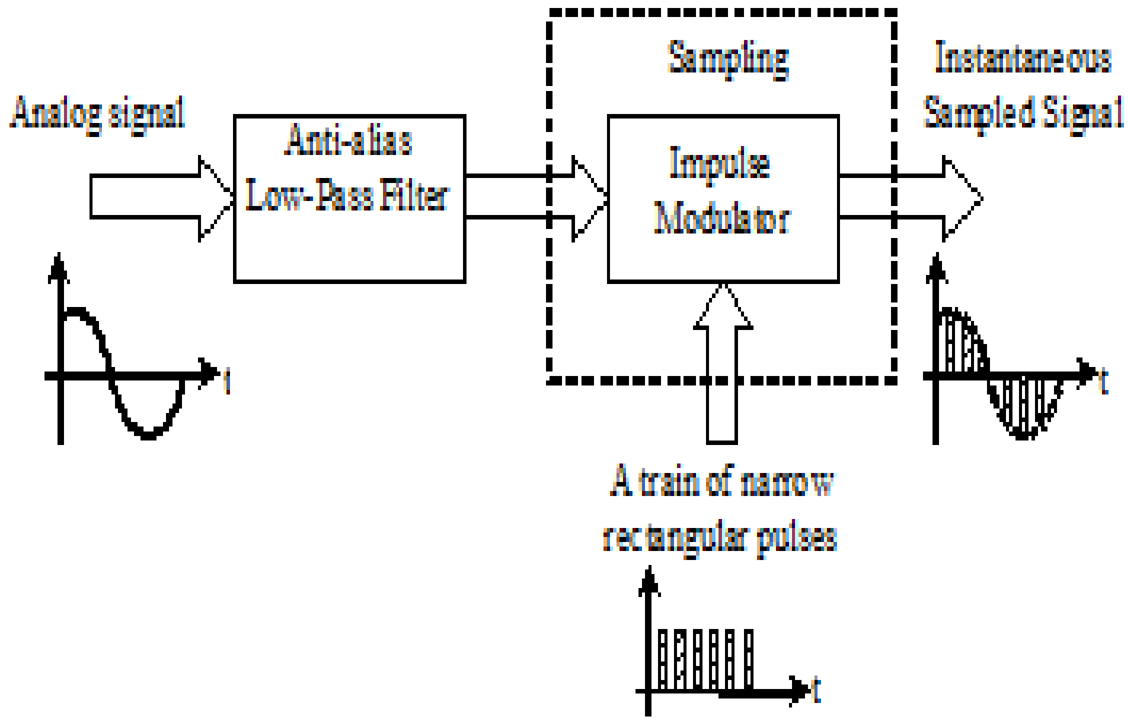
Under normal conditions, undersampling of signals causing aliasing is not recommended

Solution 1: Anti-Aliasing Analog Filter

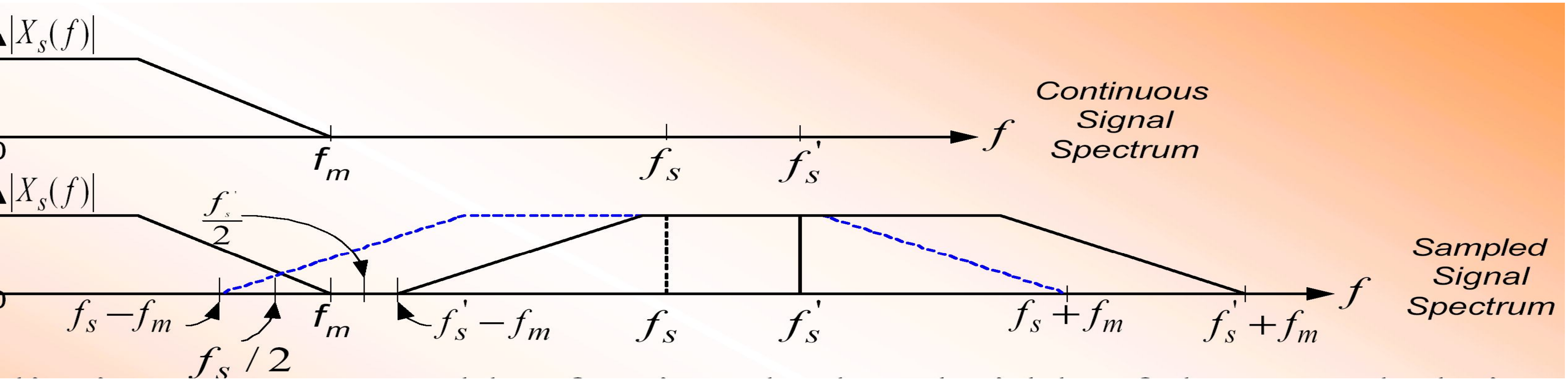
- All physically realizable signals are not completely bandlimited
- If there is a significant amount of energy in frequencies above half the sampling frequency ($f_s/2$), aliasing will occur
- Aliasing can be prevented by first passing the analog signal through an **anti-aliasing** filter (als called a **prefilter**) before sampling is performed
- The anti-aliasing filter is simply a LPF with cutoff frequency equal to half the sample rate

Antialiasing Filter

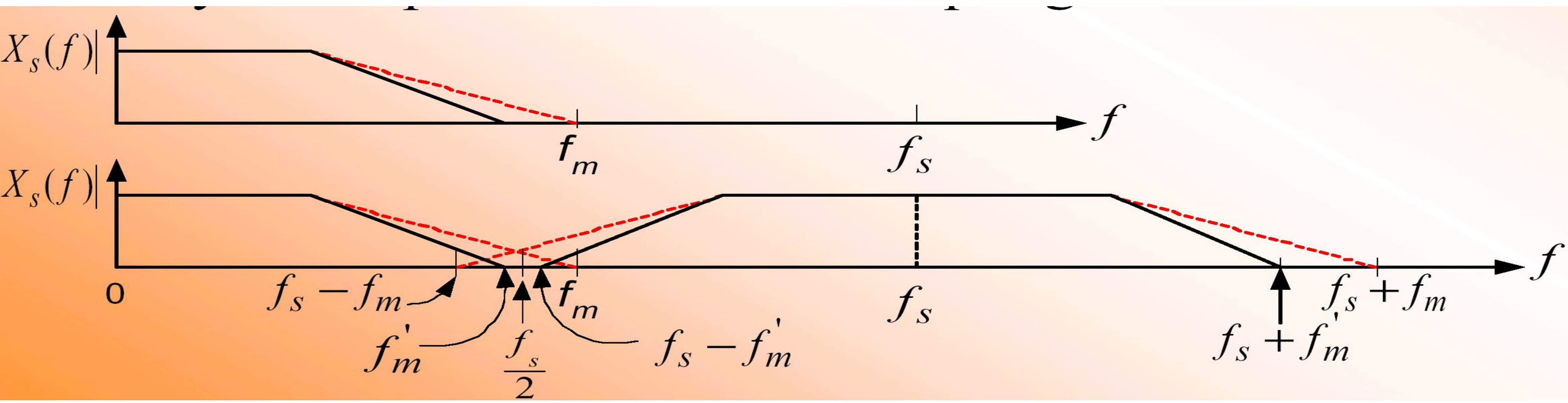
An anti-aliasing filter is a *low-pass filter* of sufficient higher order which is recommended to be used prior to sampling.



Minimizing Aliasing



- Aliasing is prevented by forcing the bandwidth of the sampled signal to satisfy the requirement of the Sampling Theorem



Solution 2: Over Sampling and Filtering in the Digital Domain

- The signal is passed through a low performance (less costly) analog low-pass filter to lim the bandwidth.
- Sample the resulting signal at a high sampling frequency.
- The digital samples are then processed by a high performance digital filter and down sample the resulting signal.

Summary Of Sampling

Ideal Sampling (or Impulse Sampling)

$$x_s(t) = x(t)x_\delta(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT$$

Natural Sampling (or Gating)

$$x_s(t) = x(t)x_p(t) = x(t)\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n}$$

Flat-Top Sampling

$$x_s(t) = x'(t) * p(t) = \left[x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)\right] * p$$

For all sampling techniques

- If *fs > 2B then we can recover x(t) exactly*
- If *fs < 2B*) ***spectral overlapping*** known as **aliasing** will occur