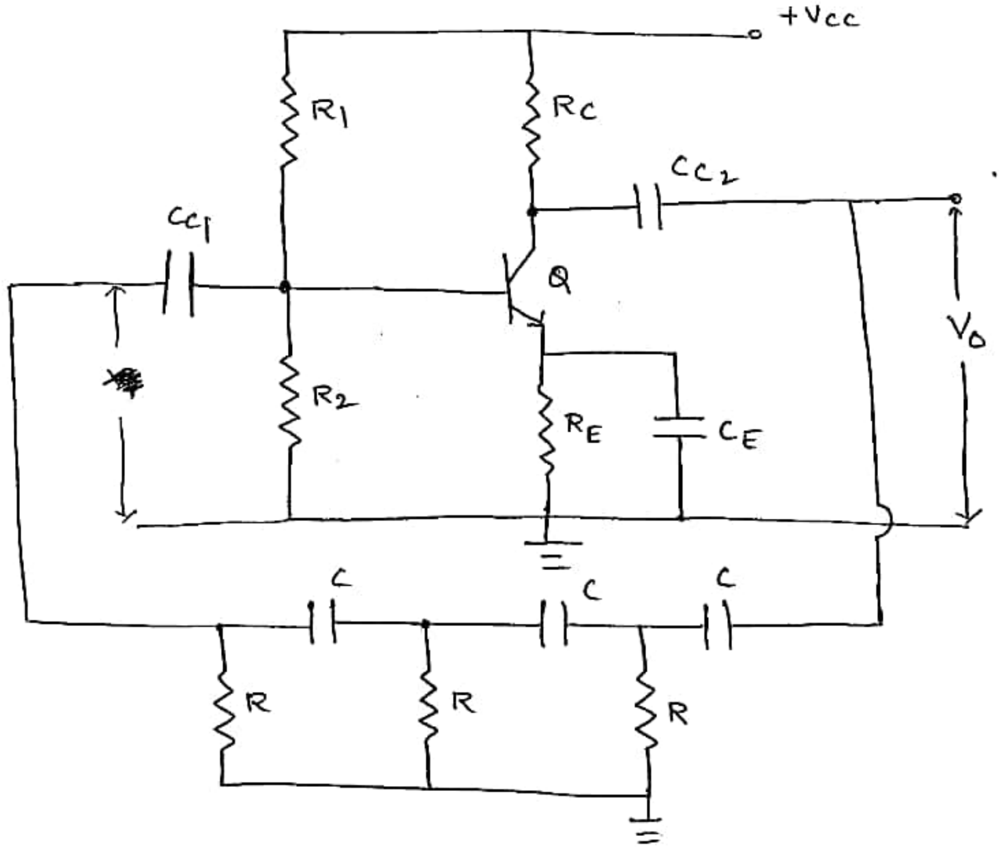
RC OSCILLATORS

All the oscillators using LC circuits operate well at high frequencies. At low frequencies, as the inductors and capacitors required for the time circuit would be very bulky. RC oscillators are found to ~~a~~ be more suitable. Two important RC oscillators are

1) RC phase shift oscillator
2) Wien Bridge oscillator.

RC phase shift oscillator:



In this oscillator the required phase shift of 180° in the feedback loop from output to input is obtained by using R and C components instead of tank circuit.

Here a common emitter amplifier is followed by three RC sections of RC phase shift n/w, the output of the last section being returned to the input.

The phase shift ϕ, given by each RC section is $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$. The value of R is adjusted such that ϕ becomes 60°. Therefore 3 RC sections produce a total phase shift of 180° between its input and output voltages for only the given frequency. Therefore at the specific frequency f_r, the total phase shift from the base of the transistor around the circuit and back to the base will be exactly 360° or 0°, thereby satisfying Barkhausen condition for oscillation.

The frequency of oscillation is given by

$$f_r = \frac{1}{2\pi RC\sqrt{6+4K}} \quad \text{where } K = \frac{R_C}{R}$$

<u>Analysis</u>: The equivalent circuit of RC phase shift oscillator using h-parameter model is shown in figure below.

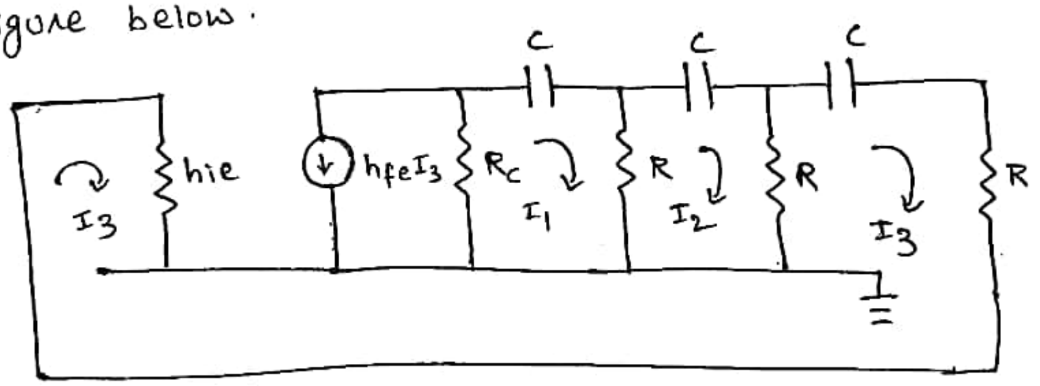


Fig: Equivalent circuit using h-parameter model.

The modified equivalent circuit is shown in figure below.

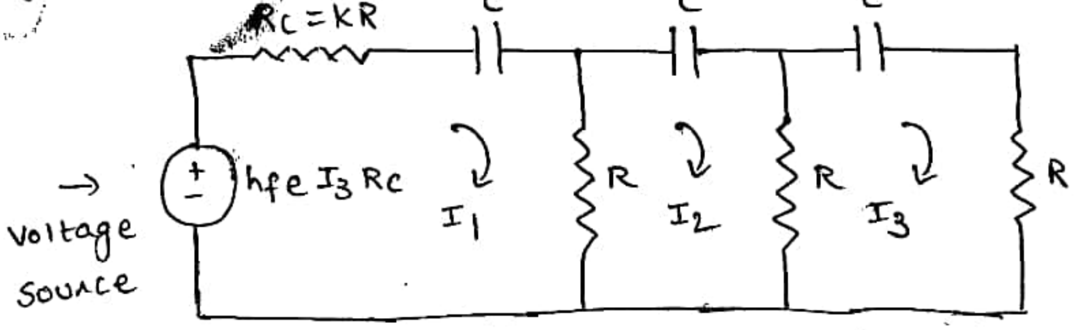


Fig: Modified Equivalent circuit.

Applying KVL for the Various loops in the modified Equivalent circuit

Loop 1:

$$-h_{fe} I_3 R_C = I_1 R_C + \frac{I_1}{j\omega C} + (I_1 - I_2) R$$

$$-h_{fe} I_3 KR = I_1 KR + \frac{I_1}{j\omega C} + I_1 R - I_2 R$$

$$-h_{fe} I_3 KR = I_1 \left(KR + R + \frac{1}{j\omega C} \right) - I_2 R$$

$$I_1 \left[(K+1) R + \frac{1}{j\omega C} \right] - I_2 R + I_3 h_{fe} K R = 0 \longrightarrow (1)$$

Loop 2 :

$$\frac{I_2}{j\omega C} + (I_2 - I_3) R + (I_2 - I_1) R = 0$$

$$-I_1 R + I_2 \left(2R + \frac{1}{j\omega C} \right) - I_3 R = 0 \longrightarrow (2)$$

Loop 3 :

$$\frac{I_3}{j\omega C} + I_3 R + (I_3 - I_2) R = 0$$

$$-I_2 R + I_3 \left(2R + \frac{1}{j\omega C} \right) = 0 \longrightarrow (3)$$

Solving the Equations (1), (2) and (3)

$$\begin{vmatrix} (K+1) R + \frac{1}{j\omega C} & -R & h_{fe} K R \\ -R & 2R + \frac{1}{j\omega C} & -R \\ 0 & -R & 2R + \frac{1}{j\omega C} \end{vmatrix} = 0$$

$$\Rightarrow \left[(K+1)R + \frac{1}{j\omega C}\right]\left[\left(2R+\frac{1}{j\omega C}\right)^2 - R^2\right] + R\left[-R\left(2R+\frac{1}{j\omega C}\right)\right] +$$

$$h_{fe} KR(R^2) = 0$$

$$\Rightarrow \left[(K+1)R + \frac{1}{j\omega C}\right]\left(4R^2 - \frac{1}{\omega^2 C^2} + \frac{4R}{j\omega C} - R^2\right) - 2R^3 - \frac{R^2}{j\omega C} +$$

$$h_{fe} KR^3 = 0$$

$$\Rightarrow 3R^3(K+1) + \frac{3R^2}{j\omega C} - \frac{(K+1)R}{\omega^2 C^2} - \frac{1}{j\omega^3 C^3} + \frac{4R^2(K+1)}{j\omega C}$$

$$- \frac{4R}{\omega^2 C^2} - 2R^3 - \frac{R^2}{j\omega C} + h_{fe} KR^3 = 0$$

$$\Rightarrow R^3(3K+1+h_{fe}K) - \left(\frac{(K+1)R+4R}{\omega^2 C^2}\right) + j\left[\frac{-4(K+1)R^2}{\omega C}\right.$$

$$\left. - \frac{2R^2}{\omega C} + \frac{1}{\omega^3 C^3}\right] = 0 \longrightarrow (4)$$

To get frequency of oscillation f_r, the imaginary part is made equal to zero

$$\frac{-4(K+1)R^2}{\omega_r C} - \frac{2R^2}{\omega_r C} + \frac{1}{\omega_r^3 C^3} = 0$$

$$-4KR^2\omega_r^2C^2 - 4R^2\omega_r^2C^2 - 2R^2\omega_r^2C^2 + 1 = 0$$

$$1 = 6R^2\omega_r^2C^2 + 4KR^2\omega_r^2C^2$$

$$\frac{1}{\omega_r^2} = (4K+6)R^2C^2$$

$$\omega_r = \frac{1}{RC\sqrt{6+4K}} \Rightarrow \boxed{f_r = \frac{1}{2\pi RC\sqrt{6+4K}}}$$

where $K = R_C/R$

The condition for maintainence of oscillation is obtained by equating real part to zero

$$R^3(3K+1+h_{fe}K) - \left(\frac{(K+1)R+4R}{\omega_n^2 C^2}\right) = 0$$

$$R^3(3K+1+h_{fe}K)\,\omega_n^2 C^2 - KR - 5R = 0$$

$$R^3(3K+1+h_{fe}K)\frac{1}{(6+4K)R^2C^2} \times C^2 = (K+5)R$$

$$3K+1+h_{fe}K = (6+4K)(5+K)$$

$$3K+1+h_{fe}K = 30+26K+4K^2$$

$$4K^2+23K+29 = h_{fe}K$$

$$h_{fe} = 4K+23+\frac{29}{K}$$

If $K=1$, then $\boxed{h_{fe} = 56}$

To find the minimum value of h_{fe} for the oscillator

$$\frac{dh_{fe}}{dK} = 0$$

$$\frac{d}{dK}\left[4K+23+\frac{29}{K}\right] = 0$$

$$4 - \frac{29}{K^2} = 0 \Rightarrow K = 2.69$$

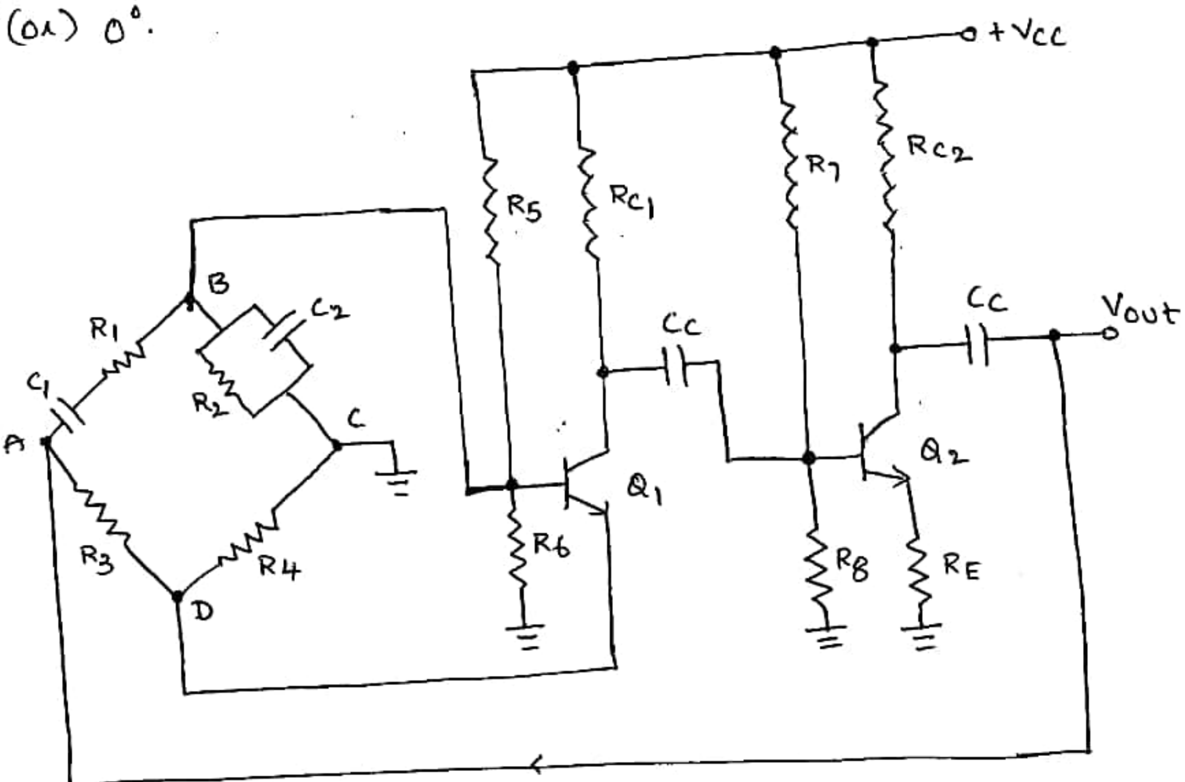
$$h_{fe(min)} = 4(2.69)+23+\frac{29}{2.69} = 44.54.$$

Thus for the circuit to oscillate we must select a transistor whose hfe should be greater than 44.54

$$\therefore \boxed{h_{fe} \geq 44.54}$$

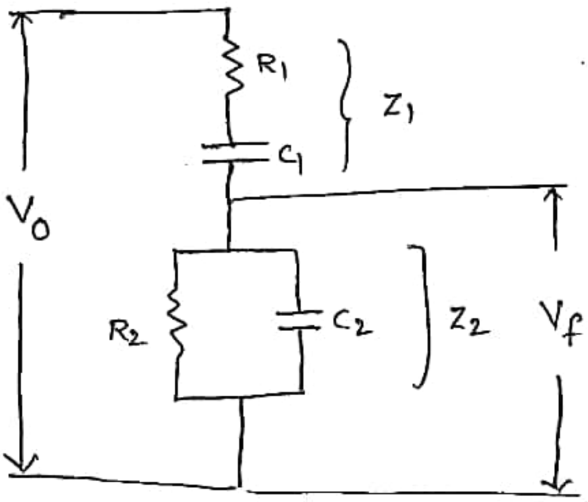
Wien Bridge oscillator:

Figure shows the circuit of a wien-bridge oscillator. The circuit consists of two-stage RC coupled amplifier which provides a phase shift of 360° (or) 0°.



feed back signal

A balanced bridge is used as the feed back network which has no need to provide additional phase shift. The feedback network consists of a lead-lag network (R_1-C_1 and R_2-C_2) and a voltage divider ($R_3 - R_4$). The lead - lag network provides a positive feedback to the input of the first stage and the voltage divider provides a negative feedback to the emitter of Q_1.



From the circuit

$$z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1} \longrightarrow (1)$$

$$z_2 = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega C_2 R_2} \longrightarrow (2)$$

$$\beta = \frac{V_f}{V_o}$$

From the above circuit

$$V_f = V_o \frac{Z_2}{Z_1 + Z_2}$$

$$\beta = \frac{V_f}{V_o} = \frac{Z_2}{Z_1 + Z_2} = \frac{\dfrac{R_2}{1 + j\omega C_2 R_2}}{\dfrac{1 + j\omega R_1 C_1}{j\omega C_1} + \dfrac{R_2}{1 + j\omega C_2 R_2}}$$

$$\beta = \frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{\dfrac{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)+j\omega R_2 C_1}{(j\omega C_1)(1+j\omega C_2 R_2)}}$$

$$\beta = \frac{j\omega R_2 C_1}{1+j\omega R_1 C_1 + j\omega R_2 C_2 - \omega^2 R_1 R_2 C_1 C_2 + j\omega R_2 C_1}$$

$$\Rightarrow \beta = \frac{j\omega R_2 C_1}{(1-\omega^2 R_1 R_2 C_1 C_2) + j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1} \rightarrow (3)$$

$$\beta = \frac{j\omega R_2 C_1}{(1-\omega^2 R_1 R_2 C_1 C_2) + j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)} \times \frac{(1-\omega^2 R_1 R_2 C_1 C_2) - j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}{(1-\omega^2 R_1 R_2 C_1 C_2) - j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}$$

$$\beta = \frac{j(\omega R_2 C_1 - \omega^3 R_1 R_2^2 C_1^2 C_2) + \omega R_2 C_1(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}{(1-\omega^2 R_1 R_2 C_1 C_2)^2 + (\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)^2}$$

To find the frequency of oscillator make imaginary part is zero.

$$\frac{\omega R_2 C_1 - \omega^3 R_1 R_2^2 C_1^2 C_2}{(1-\omega^2 R_1 R_2 C_1 C_2)^2 + (\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)^2} = 0$$

$$\omega_r R_2 C_1 = \omega_r^3 R_1 R_2^2 C_1^2 C_2$$

$$1 = \omega_r^2 R_1 R_2 C_1 C_2$$

$$\omega_r^2 = \frac{1}{R_1 R_2 C_1 C_2} \longrightarrow (4)$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \longrightarrow (5)$$

$$f_n = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \longrightarrow (6)$$

If $R_1 = R_2 = R$; $C_1 = C_2 = C$;

$$f_n = \frac{1}{2\pi\sqrt{R^2 C^2}} = \frac{1}{2\pi RC} \longrightarrow (7)$$

$\omega_n = \frac{1}{RC}$, substitute in (3)

$$\beta = \frac{j\frac{1}{RC} R_2 C_1}{(1-1) + j\left(\frac{1}{RC} R_1 C_1 + \frac{1}{RC} R_2 C_2 + \frac{1}{RC} R_2 C_1\right)}$$

Her $R_1 = R_2 = R$; $C_1 = C_2 = C$

$$\beta = \frac{j}{j(1+1+1)} = \frac{1}{3}$$

~~the~~ For sustained oscillations

$$A\beta = 1$$

$$A = 3$$

The minimum value of voltage gain required for sustained oscillation is 3

$$\therefore \boxed{A \geqslant 3}$$