IIR:- have zeros & poles

FIR:- have only zeros.

→ Discrete fourier Transform :-

If $x(n)$ is DTS, its N pt DFT is given as-

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}\cdot n\cdot k}$$

where $K = 0, 1, 2, \ldots, N-1$.

and N pt IDFT is given as-

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}\cdot n\cdot k}$$

where $n = 0, 1, \ldots, N-1$

$$\boxed{e^{-j\frac{2\pi}{N}} = W_N.}$$

It is k/a. "TWIDDLE FACTOR" or PHASE FACTOR.

In terms of twiddle factor.

DFT is -

$$X(k) = \sum_{n=0}^{N-1} x(n)\cdot W_N^{nk}.$$

where, $k = 0, 1, \ldots, N-1$

and IDFT is -

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k)\, W_N^{-nk}$$

Q. Determine two pt DFT of sequence

$x(n) = \{1, -1\}$

Here $N = 2$.

$$X(K) = \sum_{n=0}^{1} x(n)\, e^{-j\frac{2\pi}{N}\cdot n\cdot k}.$$

and $K = 0, 1$.

$$\therefore X(K) = \cancel{1\cdot e^{-j\frac{2\pi}{2}\cdot n\cdot 0} + (-1)e^{-j\frac{2\pi}{2}\cdot n\cdot 1}}$$

$$\cancel{= 1 - e^{-j\pi n}}$$

$$X(0) = x(0)\cdot e^{-j\pi\cdot 0} + x(1)\, e^{-0}.$$

$$= 1 + (-1) = 0.$$

$X(0) = 0$

$$X(1) = x(0)\cdot e^{-j2\pi/2\cdot 0\cdot 1} + x(1)\cdot e^{-j2\pi/2\cdot 1\cdot 1}$$

$$= 1 - e^{-j\pi}$$

$$= 1 - (\cos\pi - j\sin\pi)$$

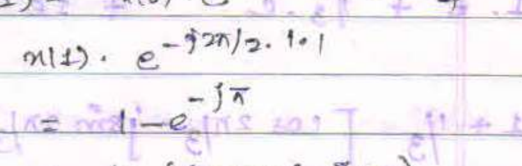
$X(1) = 2$

$\therefore X(0) = 0$ for $K = 0$.

$X(1) = 2$ for $K = 1$.

$X(K) = \{0, 2\}$

$\angle X(K) = \{0, 0\}$



Two pt DFT

1st sample ⇒ sum of x(0) + x(1)

2nd sample ⇒ diff. b/w x(0) & x(1)

Q. Determine three point DFT of $x(n) = \{\underset{0}{1}, \underset{1}{1/3}, \underset{2}{1}\}$.

Here, $N = 3$, $K = 0, 1, 2$.

$$\therefore X(k) = \sum_{n=0}^{2} x(n)\cdot e^{-j\frac{2\pi}{N}\cdot n\cdot k}$$

$$X(0) = x(0)\cdot e^{-j\frac{2\pi}{3}\cdot 0\cdot 0} + x(1)\cdot e^{-j\frac{2\pi}{3}\cdot 1\cdot 0} + x(2)\cdot e^{-j\frac{2\pi}{3}\cdot 2\cdot 0}$$

$$= 1\cdot e^{0} + 1/3\, e^{0} + 1\cdot e^{0}$$

$$X(0) = 2 + 1/3 = 7/3.$$

$$X(1) = x(0)\cdot e^{-j2\pi/3\cdot 1\cdot 0} + x(1)\cdot e^{-j\frac{2\pi}{3}\cdot 1\cdot 1} + x(2)\cdot e^{-j\frac{2\pi}{3}\cdot 1\cdot 2}$$

$$= 1\cdot e^{0} + 1/3\cdot e^{-j2\pi/3} + 1\, e^{-j4\pi/3}$$

$$= 1 + 1/3\left[\cos 2\pi/3 - j\sin 2\pi/3\right] + 1\left[\cos 4\pi/3 - j\sin 4\pi/3\right]$$

$$= 1 + 1/3\left[-1/2 - j\right.$$

$$X(1) = 1/3 + \sqrt{3}/3\, j$$

$$X(2) = x(0)\cdot e^{-j\frac{2\pi}{3}\cdot 0\cdot 2} + x(1)\cdot e^{-j\frac{2\pi}{3}\cdot 1\cdot 2} + x(2)\cdot e^{-j2\pi/3\cdot 2\cdot 2}$$

$$= 1 + 1/3\, e^{-4\pi/3\, j} + 1\, e^{-8\pi/3\, j}$$

$$= 1 + \left[\cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3}\right] 1/3 + 1\left[\cos\frac{8\pi}{3} - j\sin\frac{8\pi}{3}\right]$$

$$X(2) = 1 + \left[-1/2 + j\frac{\sqrt{3}}{2}\right] 1/3 + \left[-1/2 + j\sqrt{3}/2\right]$$

$$= 1 - 1/6 - \frac{\sqrt{3}}{3\cdot 2} j - 1/2 + \frac{\sqrt{3}}{2} j$$

$$= \frac{6-1-3}{6} - \frac{\sqrt{3}}{2} j\left[\frac{1}{3} - 1\right]$$

$$= \frac{2}{6} - \frac{\sqrt{3}}{2} j\left[\frac{-2}{3}\right]$$

$$X(2) = 1/3 - \frac{\sqrt{3}}{3} j$$

$\therefore X(0) = 7/3$.

$X(1) = 1/3 + j/\sqrt{3}$

$X(2) = 1/3 - 1/\sqrt{3}\, j$

Three point DFT of $x(n) = \{1, 1/3, 1\}$ is —

$X(0) = 7/3 \angle 0°$

$$X(1) = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} \angle \tan^{-1}\left(\frac{3}{\sqrt{3}}\right)$$

$$X(2) = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} \angle \tan^{-1}\left(\frac{-3}{\sqrt{3}}\right)$$

Q. Determine four point DFT of

$x(n) = \{x(0), x(1), x(2), x(3)\}$

$x(n) = \cos n\pi/4$.

$\therefore x(n) = \left\{1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right\}$.

$\therefore \; X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N} \cdot n \cdot K}$.

$N = 3, \quad K = 0, 1, 2, 3$.

$X(0) = x(0) \cdot e^{-j\frac{2\pi}{4} \cdot 0 \cdot 0} + x(1) \cdot e^{-j\frac{2\pi}{4} \cdot 1 \cdot 0} + x(2) \cdot e^{-j\frac{2\pi}{4} \cdot 2 \cdot 0} + x(3) \cdot e^{-j\frac{2\pi}{4} \cdot 3 \cdot 0}$

$= 1 e^{0} + \frac{1}{\sqrt{2}} e^{0} + 0 - \frac{1}{\sqrt{2}} e^{0}$

$X(0) = 1$.

$X(1) = x(0) \cdot e^{-j\frac{2\pi}{4} \cdot 1 \cdot 0} + x(1) \cdot e^{-j\frac{2\pi}{4} \cdot 1 \cdot 1} + x(2) \cdot e^{-j\frac{2\pi}{4} \cdot 2 \cdot 1} + x(3) \cdot e^{-\frac{2\pi}{4} \cdot 1 \cdot 3}$

$= 1 + \frac{1}{\sqrt{2}} \cdot e^{-\pi/2 j} + 0 - \frac{1}{\sqrt{2}} e^{-3\pi/4 j}$

$= 1 + \frac{1}{\sqrt{2}} \left[\cos\frac{\pi}{2} - j \sin\frac{\pi}{2} \right] - \frac{1}{\sqrt{2}} \left[\cos\frac{3\pi}{2} - j \sin\frac{3\pi}{2} \right]$

$= 1 + \frac{1}{\sqrt{2}} [0 - j] - \frac{1}{\sqrt{2}} [0 + j]$

$= 1 - \frac{1}{\sqrt{2}} j - \frac{1}{\sqrt{2}} j = 1 - \sqrt{2} j$

$X(1) = 1 - \sqrt{2} j$

$X(2) = x(0) \cdot e^{-j\frac{2\pi}{4} \cdot 2 \cdot 0} + x(1) e^{-j\frac{2\pi}{4} \cdot 2 \cdot 1} + x(2) e^{-j2\pi/4 \cdot 2 \cdot 2} + x(3) e^{-j2\pi/4 \cdot 2 \cdot 3}$

$= 1 + \frac{1}{\sqrt{2}} e^{-\pi j} - \frac{1}{\sqrt{2}} e^{-3\pi j}$

$= 1 + \frac{1}{\sqrt{2}} [\cos\pi - j \sin\pi] - \frac{1}{\sqrt{2}} [\cos 3\pi - j \sin 3\pi]$

$= 1 + \frac{1}{\sqrt{2}} [-1 - 0] - \frac{1}{\sqrt{2}} [-1 - 0]$

$X(2) = 1$

$X(3) = x(0) \cdot e^{-j\frac{2\pi}{4} \cdot 3 \cdot 0} + x(1) \cdot e^{-j\frac{2\pi}{4} \cdot 3 \cdot 1} + x(2) \cdot e^{-j\frac{2\pi}{4} \cdot 3 \cdot 2} + x(3) \cdot e^{-j\frac{2\pi}{4} \cdot 3 \cdot 3}$.

$= 1 + \frac{1}{\sqrt{2}} e^{-3/2 \pi j} - \frac{1}{\sqrt{2}} e^{-9\pi/2 j}$

$= 1 + \frac{1}{\sqrt{2}} \left[\cos\frac{3\pi}{2} - j \sin\frac{3\pi}{2} \right] - \frac{1}{\sqrt{2}} \left[\cos\frac{9\pi}{2} - j \sin\frac{9\pi}{2} \right]$.

$X(2) = 1 + \sqrt{2} j$

$\therefore \; X(K) = \{ 1, \; 1 - \sqrt{2} j, \; 1, \; 1 + \sqrt{2} j \}$

Deduction -

$\therefore$ In terms of twiddle factor.

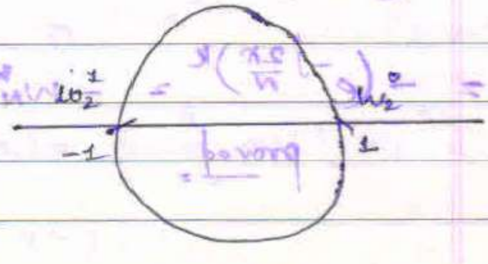
$X(K) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{n \cdot k}$.

$X(0) = x(0) + x(1) \cdot \ldots + x(3)$

$X(1) = x(0) + x(1) w^1 + x(2) w^2 + x(3) w^3$.

$X(2) = x(0) + x(1) w^2 + x(2) w^4 + x(3) w^6$.

$W_N = 1 \cdot e^{-j\frac{2\pi}{N}} = r e^{j\theta}$.



• properties of Twiddle factor :-

1. Periodicity :-

It is always periodic fun.

$$W_N^{(K+N)} = W_N^{K}.$$

$$W_N^{(K+rN)} = W_N^{K}.$$

proof :- let $W_N = e^{-j\frac{2\pi}{N}}$

$$\therefore \left(e^{-j\frac{2\pi}{N}}\right)^{(k+rN)} = e^{-j\frac{2\pi}{N}\cdot k} \cdot e^{-j\frac{2\pi}{N}\cdot N\cdot r}$$

$$= e^{-j\frac{2\pi}{N}\cdot k} \cdot \left(e^{-j2\pi}\right)^{r\cdot N}$$

$$= e^{-j2\pi/N\, k} = W_N^{k}.$$

proved.

2. Half Periodicity :-

$$W_N^{(K+N/2)} = -W_N^{K}.$$

proof :- $W_N = e^{-j\frac{2\pi}{N}}$

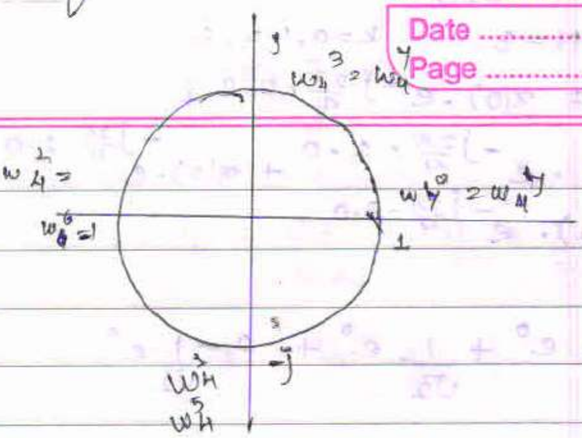
$$\therefore \left(e^{-j\frac{2\pi}{N}}\right)^{(k+N/2)} = \left(e^{-j\frac{2\pi}{N}}\right)^{k} \cdot \left(e^{-j\frac{2\pi}{N}}\right)^{N/2}$$

$$= \left(e^{-j\frac{2\pi}{N}}\right)^{k} \cdot (-1)$$

$$= -\left(e^{-j\frac{2\pi}{N}}\right)^{k} = -W_N^{k}.$$

proved.

for N=4.



for half periodicity

$$W_4^3 = W_4^{(1+4/2)} = -W_4^1$$

$$W_4^2 = W_4^{(0+4/2)} = -W_4^0$$

3. $W_N^N = 1$; $W_N^{N/2} = -1$

Note:- Shortcut method.

let $x(n) = \left\{1, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}$

$$X(k) = \sum_{n=0}^{3} x(n)\cdot W_4^{nk}.$$

$\therefore k = 0, 1, 2, 3$

$$X(0) = x(0) + x(1) + x(2) + x(3) = 1$$

$$X(1) = \sum_{n=0}^{3} x(n)\cdot W_4^{n}.$$

$$= x(0) + x(1)W_4^{1} + x(2)W_4^{2} + x(3)W_4^{3}.$$

$$= 1 + \frac{1}{\sqrt{2}}(-1) + 0\times(-1) + \left(-\frac{1}{\sqrt{2}}\right)j$$

$X(2) = x(0) + x(1)\cdot w_4^2 + x(2) W_4^4 + x(3)\cdot w_4^6$

$= 1 + \frac{1}{\sqrt{2}}(-1) + 0 + \left(\frac{-1}{\sqrt{2}}\right)(-1)$

$= 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 1$

$X(3) = x(0) + x(1)\, w_4^3 + x(2)\, w_4^6 + x(3)\cdot w_4^9$

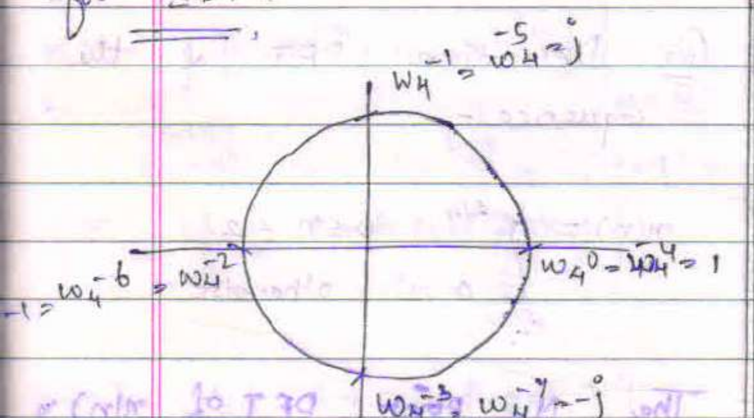
$= 1 + \frac{1}{\sqrt{2}}(j) + 0 + \left(\frac{-1}{\sqrt{2}}\right)(-j)$

$= 1 + \frac{j}{\sqrt{2}} + \frac{j}{\sqrt{2}} = 1 + \sqrt{2}j$

$X(K) = \{1,\ 1-\sqrt{2}j,\ 1,\ 1+\sqrt{2}j\}$

Note :- for DFT → clockwise
IDFT → anticlockwise

for IDFT



$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} x(K)\cdot W_N^{-nK}.$$

$n = 0, 1, 2, \ldots$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(K)\cdot W_N^{-nk}.$$

$x(0) = \frac{1}{4} \sum_{n=0}^{3} x(n)\, w_4^0$

$= \frac{1}{4}\left[x(0) + x(1) + x(2) + x(3)\right] 1.$

$= \frac{1}{4}\left[1 + (1-\cancel{\sqrt{2}j}) + 1 + (1+\cancel{\sqrt{2}j})\right]$

$= \frac{3}{4}.$

$x(1) = \frac{1}{4} \sum_{k=0}^{3} x(k)\cdot w_4^{-k}.$

$= \frac{1}{4}\left[X(0)\, W_4^{-0} + X(1)\cdot W_4^{-1} + X(2)\, W_4^{-2} + X(3)\, W_4^{-3}\right]$

$= \frac{1}{4}\left[1.1 + (1-\sqrt{2}j)\, j + 1.(-1) + (1+\sqrt{2}j).(-j)\right]$

$= \frac{1}{4}\left[\cancel{1} + \cancel{j} + \sqrt{2} - \cancel{1} - \cancel{j} + \sqrt{2}\right]$

$= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = 1/\sqrt{2}.$

$x(2) = \frac{1}{4} \sum_{K=0}^{3} x(k)\cdot w_4^{-2k}.$

$= \frac{1}{4}\left[x(0)\cdot w_4^{-0} + X(1)\cdot w_4^{-2} + X(2)\cdot w_4^{-4} + X(3)\cdot w_4^{-6}\right]$

$= \frac{1}{4}\left[1.(1) + (1-\sqrt{2}j)(-1) + 1(-j) + (1+\sqrt{2}j)(-1)\right]$

$x(2) = 0$

$x(3) = \frac{1}{4} \sum_{k=0}^{3} x(k)\cdot w_4^{-3k}.$

$x(3) = 1/\sqrt{2}.$