Q. Find the z-transform of unit impulse sequence $\delta(n)$

$$f(n) = \delta(n)$$

$$F(z) = \sum_{n=-\infty}^{\infty} f(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \delta(n) \cdot z^{-n}$$

Q.

$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$

$1 - x + x^2 - x^3 + \cdots = \frac{1}{1+x}$

$1 + 2x + 3x^2 + 4x^3 + \cdots = \frac{1}{(1-x)^2}$

$1 - 2x + 3x^2 - 4x^3 + \cdots = \frac{1}{(1+x)^2}$

Q.

find z transform of $f(n) = e^{-j\omega n} u(n)$

$$F(z) = \sum_{n=-\infty}^{\infty} f(n) z^{-n}$$

$$F(z) = \sum_{n=-\infty}^{\infty} e^{-j\omega n} u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} e^{-j\omega n} z^{-n}$$

$$\cancel{= \sum_{n=0}^{\infty} e^{-j\omega - 1}}$$

$$= \sum_{n=0}^{\infty} (e^{-j\omega} \cdot z^{-1})^n$$

$$= (e^{-j\omega} \cdot z^{-1})^0 + (e^{-j\omega} \cdot z^{-1})^1 + (e^{-j\omega} \cdot z^{-1})^2 + \cdots$$

$$F(z) = \frac{1}{1 - (e^{-j\omega} \cdot z^{-1})}$$


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$$F(z) = \frac{z}{z - e^{-j\omega}}$$

$$\left|\left(e^{-j\omega} z^{-1}\right)\right| < 1$$

$$|z^{-1}| < 1$$

$$|z| > 1$$

Q. Find the z-transform of $f(n) = \sin\omega_0 n \; u(n)$

$$F(z) = \sum_{n=-\infty}^{\infty} \sin\omega_0 n \; u(n) \; z^{-n}$$

$$= \sum_{n=0}^{\infty} \sin\omega_0 n \; z^{-n}$$

$$\sin\omega_0 n = \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}$$

$$= \frac{1}{2j}\left[\sum_{n=0}^{\infty} \left\{ e^{j\omega_0 n} - e^{-j\omega_0 n} \right\} z^{-n}\right]$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} e^{j\omega_0 n} z^{-n} - \frac{1}{2j} \sum_{n=0}^{\infty} e^{-j\omega_0 n} z^{-n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left(e^{j\omega_0} \cdot z^{-1}\right)^n - \frac{1}{2j} \sum_{n=0}^{\infty} \left(e^{-j\omega_0} \cdot z^{-1}\right)^n$$

$$= \frac{1}{2j}\left[\left(e^{j\omega_0} \cdot z^{-1}\right)^0 + \left(e^{j\omega_0} \cdot z^{-1}\right)^1 + \cdots\right] - \frac{1}{2j}\left[\left(e^{-j\omega_0} \cdot z^{-1}\right)^0 + \left(e^{-j\omega_0} z^{-1}\right)^1 + \cdots\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1-e^{j\omega_0}z^{-1}}\right] - \frac{1}{2j}\left[\frac{1}{1-e^{-j\omega_0}z^{-1}}\right]$$

$$\cancel{= \frac{1}{2j}\left[\frac{z}{z-e^{j\omega_0}} - \frac{z}{z-e^{-j\omega_0}}\right]}$$

$$\cancel{= \frac{z}{2j}\left[\frac{z-e^{-j\omega_0} - z + e^{j\omega_0}}{(z-e^{j\omega_0})(z-e^{-j\omega_0})}\right]}$$

$$= \cancel{\frac{z}{2j}}$$

$$= \frac{1}{2j}\left[\frac{z}{z-e^{j\omega_0}} - \frac{z}{z-e^{-j\omega_0}}\right]$$

$$= \frac{z}{2j}\left[\frac{\cancel{z}-e^{-j\omega_0} - \cancel{z}+e^{j\omega_0}}{(z-e^{j\omega_0})(z-e^{-j\omega_0})}\right]$$

$$= \frac{z}{2j}\left[\frac{e^{j\omega_0} - e^{-j\omega_0}}{z^2 - ze^{-j\omega_0} - e^{j\omega_0}z + 1}\right]$$

$$\sin\omega_0 = \frac{e^{j\omega_0} - e^{-j\omega_0}}{2j}$$

$$= z\left[\frac{\sin\omega_0}{z^2 - z(e^{j\omega_0} + e^{-j\omega_0}) + 1}\right]$$

$$\cos\omega_0 = \frac{e^{j\omega_0} + e^{-j\omega_0}}{2}$$

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$$= \frac{z \sin \omega_0}{z^2 - 2z\cos\omega_0 + 1}$$

ROC $|z| > 1$


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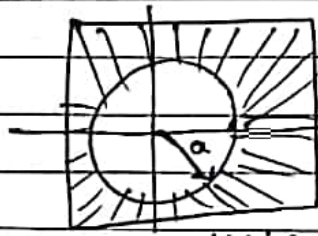
$$= z\left[F(z) - f(0)\right]$$

$$y(z) = zF(z) - zf(0)$$

Q. Find the z transform of $f(n) = a^n u(n)$

$$\sum_{n=-\infty}^{\infty} a^n \left[u(n)\right] z^{-n}$$

$$\sum_{n=0}^{\infty} a^n \cdot z^{-n}$$



$|z| > |a|$

$$\sum_{n=0}^{\infty} (a \cdot z^{-1})^n$$

$$= (a \cdot z^{-1})^0 + (a \cdot z^{-1})^1 + (a \cdot z^{-1})^2 + \cdots$$

$$= \frac{1}{1 - a \cdot z^{-1}} = \frac{z}{z-a}$$

This is a geometric series of infinite length & converges if $|a z^{-1}| < 1$ i.e., $|z| > |a|$

$$a^n u(n) \xrightarrow{z} \frac{z}{z-a}$$

ROC: $|z| > |a|$

Q. $-a^n \, u(-n-1)$

$$-\sum_{n=-\infty}^{\infty} a^n \, u(-n-1) \, z^{-n}$$

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$$= \left[ - \sum_{n=-\infty}^{-1} a^n (z^{-n}) \right]$$

$$\cancel{= - \sum_{n=-\infty}^{-1} (a^{-1} \cdot z^{+1})^{-n}}$$

$$\cancel{= \left[ (a \cdot z^{-1})^n \right]}$$

$$= - \sum_{n=-\infty}^{-1} (a \cdot z^{-1})^n$$

$$= - \sum_{n=-\infty}^{-1} (a^{-1} z)^{-n}$$

$$= - \left[ (a^{-1} \cdot z)^1 + (a^{-1} z)^2 + \dots \right]$$

$$= - \left[ \frac{a^{-1} \cdot z}{1 - a^{-1} z} \right]$$

$$= \left[ \frac{z}{a - z} \right]$$

$$\text{ROC} \quad |a^{-1} z| < 1$$

$$|z| < |a|$$

$$-a^n \cdot u(-n-1) \xrightarrow{z} \frac{z}{z-a}$$

$$\text{ROC} \quad |z| < |a|$$