System with finite & Infinite duration Response

Non Recursive System

When the o/p y(n) of the system depends on present & past i/p then it is called non - recursive system

$$y(n) = f[x(n), x(n-1) \dots x(n-m)]$$

Recursive System

When the o/p y(n) of the system depends upon the present & past i/p as well as past o/p, It is called recursive system.

The causal FIR and IIR LTI systems can be efficiently implemented using recursive systems. The recursive system are efficient in terms of memory requirement & computation.

Finite Impulse Response (FIR) system

The system for which unit

sample response h(n) has finite no. of terms. They are called FIR systems

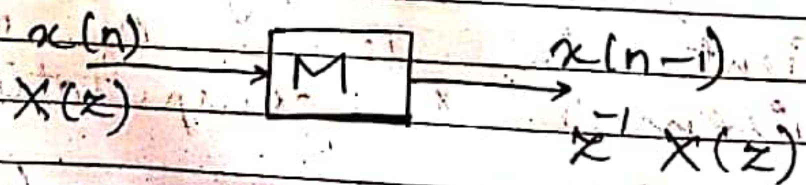
Infinite Impulse Response (IIR) System:

The system for which infinite no of unit sample response terms are to be considered, called as IIR system

The different types of structure used to realize the digital filter are as foll.

1) Direct form I
2) Direct form II
3) Cascade form structure
4) Parallel form structure

Memory Element



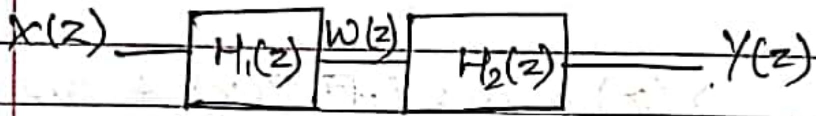
$$H(z) = \frac{z^{-1} X(z)}{X(z)}$$

$$H(z) = z^{-1}$$

Q. Develop direct form I realization of the TF $H(z) = \dfrac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$

$$H_1(z) = 3 + 3.6z^{-1} + 0.6z^{-2}$$

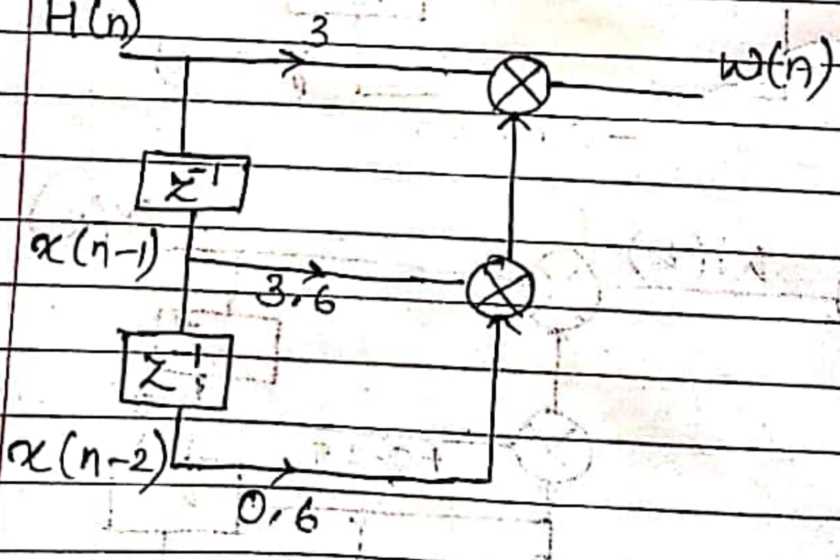
~~$h_1(n)$~~ $H_2(z) = \dfrac{1}{1 + 0.1z^{-1} - 0.2z^{-2}}$



$$H_1(z) = \frac{W(z)}{X(z)} = 3 + 3.6z^{-1} + 0.6z^{-2}$$

$$W(z) = 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z)$$

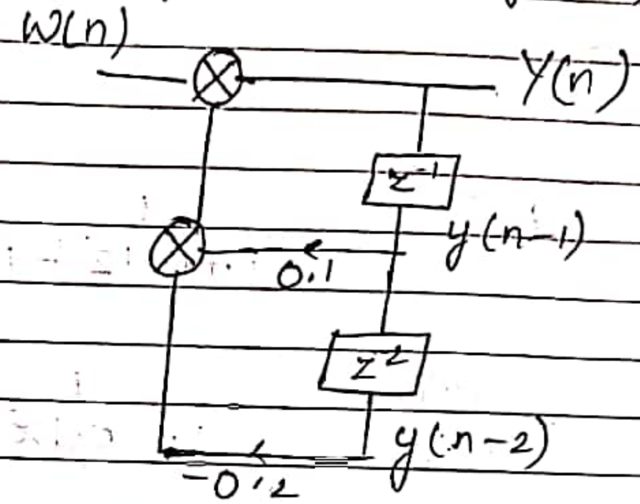
$$w(n) = 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

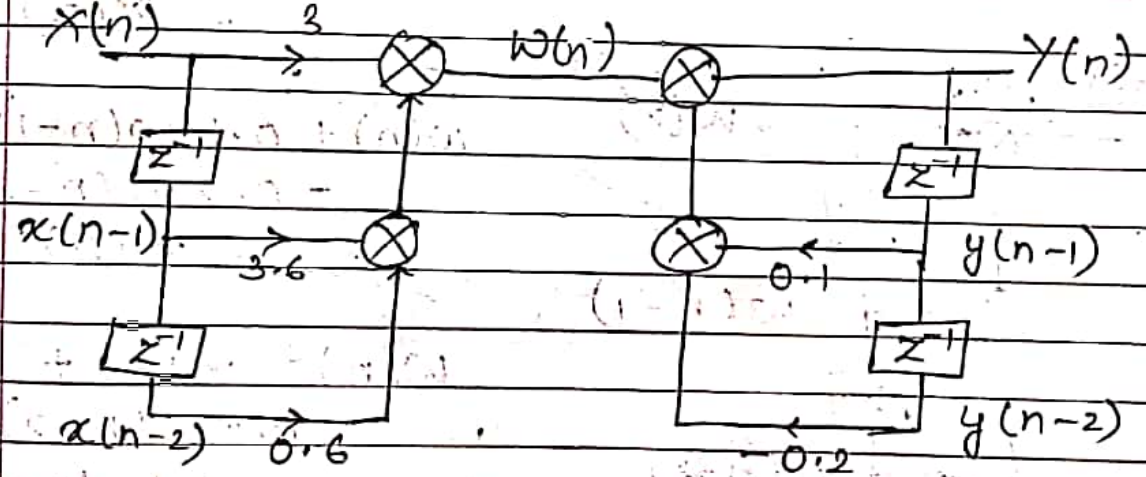


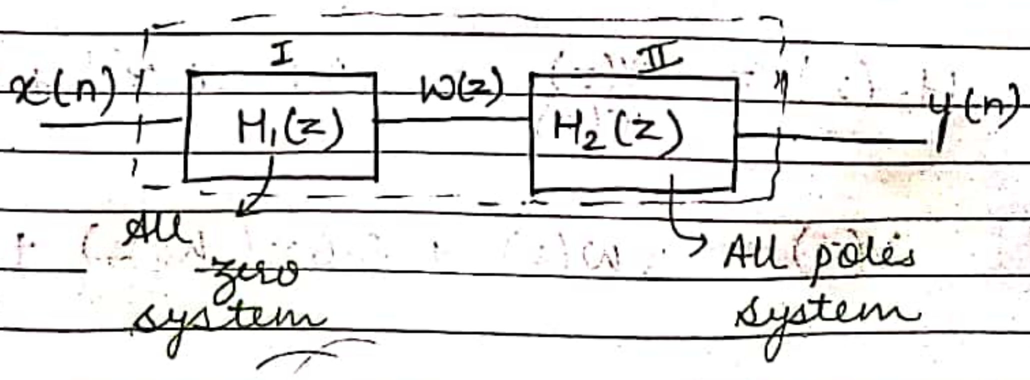
$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{1+0.1z^{-1}-0.2z^{-2}}$$

$$W(z) = Y(z) + 0.1\, z^{-1}\, Y(z) - 0.2 z^{-2}\, Y(z)$$

$$w(n) = y(n) + 0.1\, y(n-1) - 0.2\, y(n-2)$$







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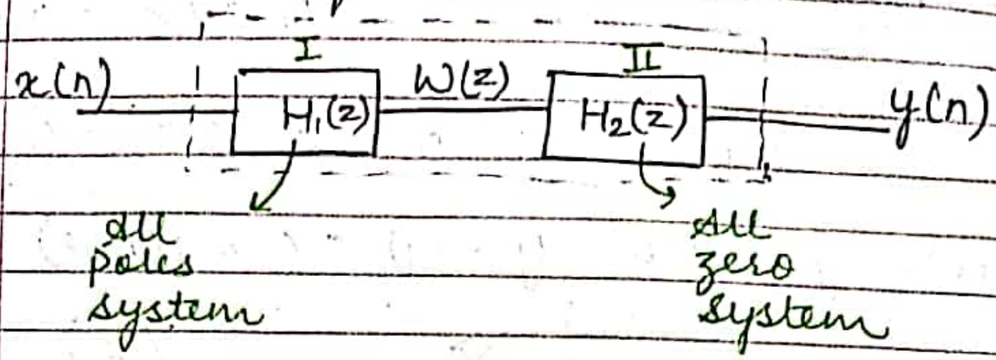
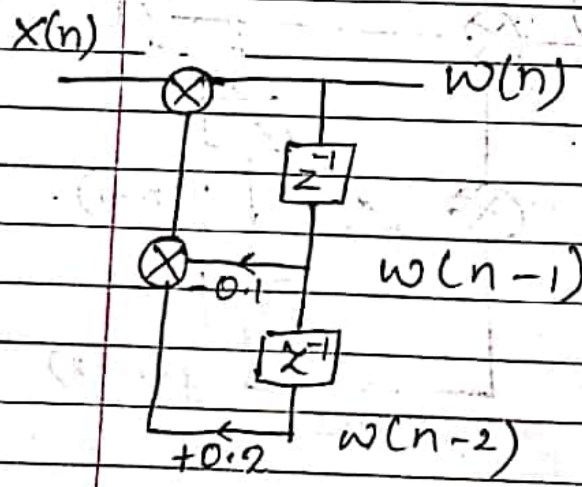
Direct form II



$$H_1(z) = \frac{1}{1+0.1z^{-1} - 0.2z^{-2}}$$

$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1+0.1z^{-1} - 0.2z^{-2}}$$

$$W(z) + 0.1z^{-1}W(z) - 0.2z^{-2}W(z) = X(z)$$



$$w(n) + 0.1\,w(n-1) - 0.2\,w(n-2) = x(n)$$

$$w(n) = x(n) + 0.2\,w(n-2) - 0.1\,w(n-1)$$

$$H_2(z) = \frac{Y(z)}{W(z)} = 3 + 3.6z^{-1} + 0.6z^{-2}$$

$$Y(z) = 3W(z) + 3.6z^{-1}W(z) + 0.6z^{-2}W(z)$$

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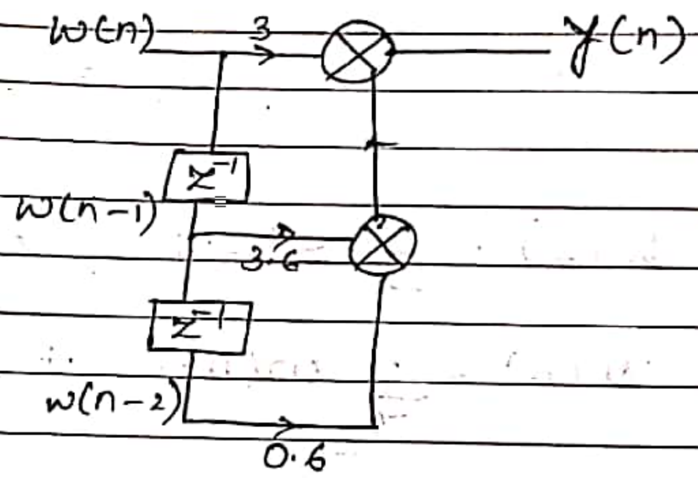
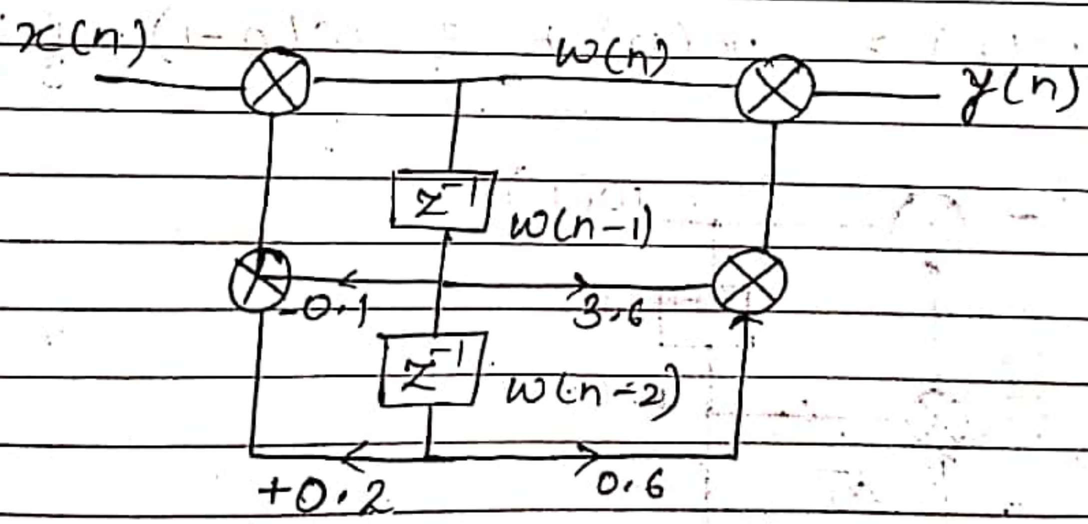
$y(n) = 3\,w(n) + 3.6\,w(n-1) + 0.6\,w(n-2)$





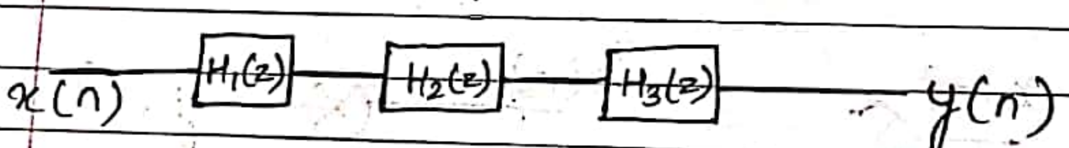
Cascade form Realization

To obtain the cascade form realization, the numerator & denominator of given transfer funct^n $H(z)$ is factored into the product of II order terms

Then the total TF. $H(z)$ is expressed as

$$H(z) = H_1(z) . H_2(z) . H_3(z) \ldots H_K(z)$$

Then each sub transfer function $[H_1(z)\ H_2(z) \ldots H_K(z)]$ may be realized using direct form I or direct form II structures.



Q. $H(z) = \dfrac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$

Draw cascade structures

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 + 0.3z^{-1})}$$

$$= H_1(z) . H_2(z)$$

$$H_1(z) = \frac{1 - z^{-1}}{1 - 0.5z^{-1}}$$

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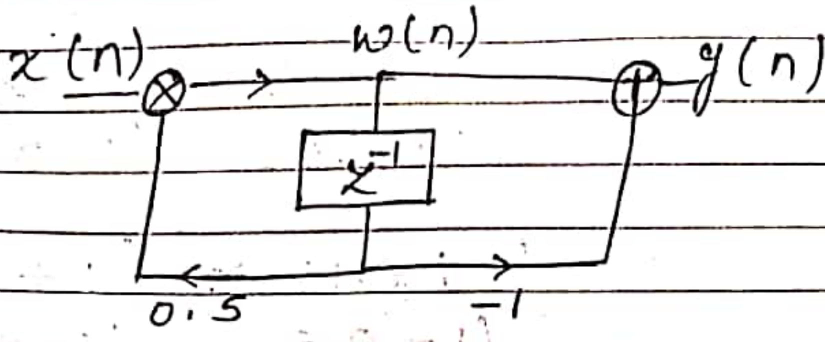
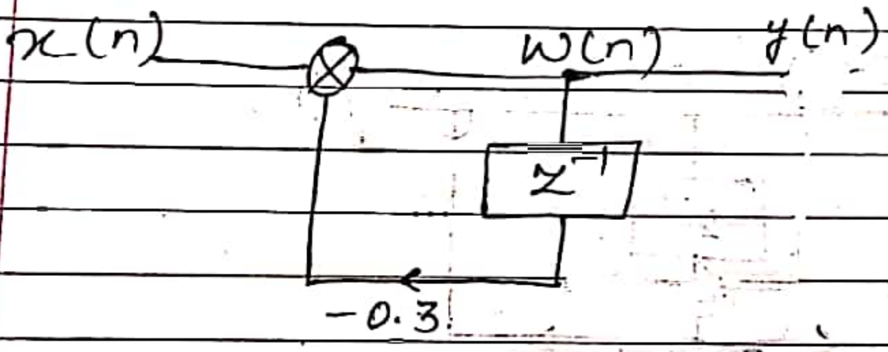
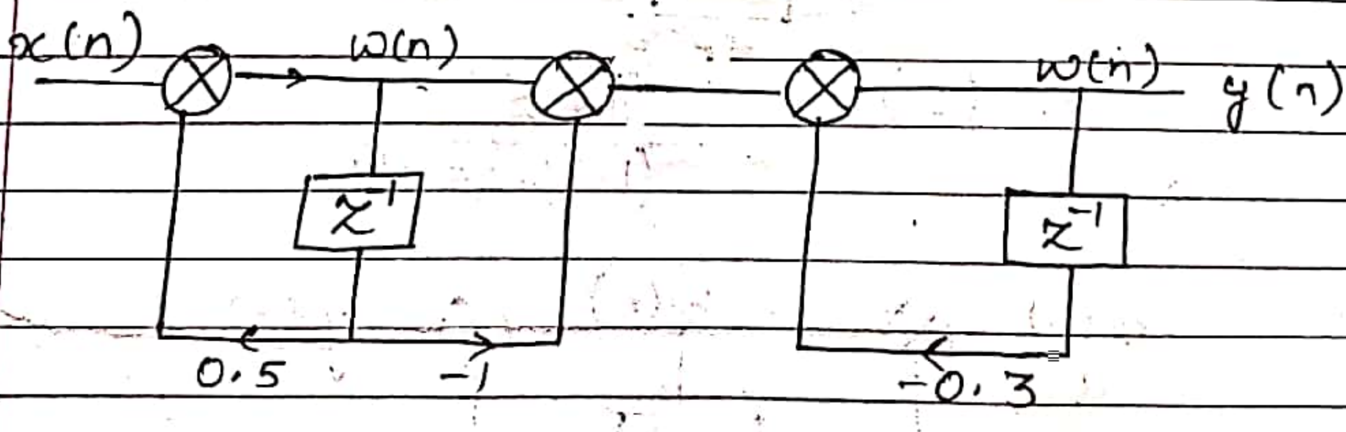


$$H_2(z) = \frac{1}{(1 + 0.3z^{-1})}$$





Q. $H(z) = \dfrac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$ draw parallel form

$= \dfrac{z(z - 1)}{z^2 - 0.2z - 0.15}$

~~$= \dfrac{A}{z^{-1}} + \dfrac{B}{z^{-2}}$~~

$\dfrac{H(z)}{z} = \dfrac{(z - 1)}{(z - 0.5)(z + 0.3)}$

$= \dfrac{A}{(z - 0.5)} + \dfrac{B}{(z + 0.3)}$

$A = -0.625$

$B = 1.625$

$$H(z) = -\frac{0.625z}{(z-0.5)} + \frac{1.625z}{(z+0.3)}$$

$$= \frac{-0.625}{1-0.5z^{-1}} + \frac{1.625}{1+0.3z^{-1}}$$

$$= H_1(z) + H_2(z)$$