UNIT- I

MICROWAVE TRANSMISSION LINES-I

INTRODUCITON

Microwaves are electromagnetic waves with frequencies between 300MHz (0.3GHz) and 300GHz in the electromagnetic spectrum.

Radio waves are electromagnetic waves within the frequencies 30KHz - 300GHz, and include microwaves. Microwaves are at the higher frequency end of the radio wave band and low frequency radio waves are at the lower frequency end.

Mobile phones, phone mast antennas (base stations), DECT cordless phones, Wi-Fi,WLAN, WiMAX and Bluetooth have carrier wave frequencies within the microwave band of the electromagnetic spectrum, and are pulsed/modulated. Most Wi-Fi computers in schools use 2.45GHz (carrier wave), the same frequency as microwave ovens. Information about the frequencies can be found in Wi-Fi exposures and guidelines.

It is worth noting that the electromagnetic spectrum is divided into different bands based on frequency. But the biological effects of electromagnetic radiation do not necessarily fit into these artificial divisions.

A waveguide consists of a hollow metallic tube of either rectangular or circular cross section used to guide electromagnetic wave. Rectangular waveguide is most commonly used as waveguide. waveguides are used at frequencies in the microwave range.

At microwave frequencies (above 1GHz to 100 GHz) the losses in the two line transmission system will be very high and hence it cannot be used at those frequencies . hence microwave signals are propagated through the waveguides in order to minimize the losses.

Properties and characteristics of waveguide:

1. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave through multiple reflections .
2. when the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall .the process results in a component of either electric or

magnetic fields in the direction of propagation of the resultant wave.

3. TEM waves cannot propagate through the waveguide since it requires an axial conductor for axial current flow .
4. when the wavelength inside the waveguide differs from that outside the guide, the velocity of wave propagation inside the waveguide must also be different from that through free space.
5. if one end of the waveguide is closed using a shorting plate and allowed a wave to propagate from other end, then there will be complete reflection of the waves resulting in standing waves.

Waveguides

A waveguide consists of a hollow metallic tube of a rectangular or circular shape used to guide an electromagnetic wave. Waveguides are used principally at frequencies in the microwave range.

In waveguide the electric and magnetic fields are confined the space with in the guides. Thus no power is lost through radiation and even the dielectric loss is negligible since the guides are normally air-filled. However, there is some power loss as heat in the walls of the guide, but the loss is very small.

It is possible to propagate several modes of EM waves with in a waveguide. These modes correspond to solutions of Maxwell's Equations for particular waveguide.

If the frequency of the impressed signal is above the cut-off frequency for a given mode, the EM energy can be transmitted through the guide for that particular mode without attenuation.

The mode which is having the lowest cut-off frequency is called the 'Dominant Mode'

Waveguide are two types

i) Rectangular waveguide ii) Circular waveguide

Rectangular Waveguide

A Rectangular waveguide is a hollow metallic tube with a rectangular cross section.

When the waves travel longitudinally down the guide because of conducting walls plane waves are reflected from wall to wall. This process results in a component of either electric or magnetic field in the direction of propagation of the resultant wave. Therefore the wave is no longer a transverse electromagnetic wave.

Any uniform plane wave in a lossless guide may be resolved into TE and TM waves.

In rectangular guide the modes are designed TE_{mn} or $TM_{mn.}$

Propagation of waves in Rectangular waveguides

Consider a rectangular waveguide situated in the rectangular coordinate system with its breadth along x-axis, width along y-axis and the wave is assume to propagate along the z-direction. Waveguide is filled with air. In a waveguide no TEM wave is exists.

TEM(Transverse Electromagnetic wave): in TEM both electric and magnetic fields are purely transverse to the direction of propagation and consequens have no 'z' directed E & H components.

TE(Transverse Electric Wave) In TE wave only the E field is purely transverse to the direction of propagation and the magnetic field is not purely transverse

i.e. E_z=0,H_z#0

TM(Transverse Magnetic Wave) In TE wave only the H field is purely transverse to the direction of propagation and the Electric field is not purely transverse

i.e. E_z#0,H_z=0

HE(Hybrid wave) In this neither electric nor magnetic fields are purely transverse to the direction of propagation.

i.e. E_z#0, H_z#0

WAVE EQUATIONS

Since we assumed that the wave direction is along z-direction then the wave equation are

$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z$ for TM wave-------(1)

$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$ for TE wave -------(2)

Where $E_z = E_0 e^{-\gamma z}$, $H_z = H_0 e^{-\gamma z}$-------------------(3)

The condition for wave propagation is that γ must be imaginary.

Differentiating eqn(3) w.r.t 'z' we get

$\partial E_z/\partial z = E_0 e^{-\gamma z}(-\gamma) = -\gamma E_z$-----------------(4)

Hence we can define operator $\partial/\partial z = -\gamma$-----------(5)

By differentiating eqn(4) w.r.t 'z' we get

$\partial^2E_z/\partial z^2 = \gamma^2E_z$

We can define the operator

$\partial^2/\partial z^2 = \gamma^2$---------(6)

From eqn(1) we can write

$\nabla^2E_z= -\omega^2\mu\epsilon E_z$

By expanding ∇^2E_z in rectangular coordinate system

$\frac{\partial 2Ez}{\partial x2} + \frac{\partial 2Ez}{\partial y2}+h^2 E_Z=0$ for TM wave---------(7)

Similarly $\frac{\partial 2Hz}{\partial x2} + \frac{\partial 2Hz}{\partial y2}+h^2 H_Z=0$ for TE wave-------------(8)

By solving above two partial differential equations we get solutions for E_z and H_z. Using Maxwell's equations. it is possible to find the various components along x an y-directions.

From Maxwell's first equation, we have

$\nabla XH= j\omega\epsilon E$

$$\begin{vmatrix} ax & ay & az \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Hx & Hy & Hz \end{vmatrix} = j\omega\epsilon[E_xa_x+E_ya_y+E_za_z]$$

$a_x\rightarrow \gamma H_y+\partial H_z/\partial y = j\omega\epsilon E_x$________________(9)

$a_y\rightarrow \gamma H_x+\partial H_z/\partial x = -j\omega\epsilon E_y$--------------------(10)

$a_z \rightarrow \partial H_y/\partial x+\partial Hx/\partial y = j\omega\epsilon E_z$---------------(11)

similarly from Maxwell's 2nd equation we have

$\nabla XE = -j\omega\mu H$

By expanding

$$\begin{vmatrix} ax & ay & az \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Ex & Ey & Ez \end{vmatrix} = -j\omega[H_xa_x+H_ya_y+H_za_z]$$

Since $\partial/\partial z =\gamma$

$$\begin{vmatrix} ax & ay & az \\ \partial/\partial x & \partial/\partial y & -\gamma \\ Ex & Ey & Ez \end{vmatrix} = -j\omega[H_xa_x+H_ya_y+H_za_z]$$

By comparing a_x,a_y,a_z components

$a_x\rightarrow \gamma E_y+\partial Ez/\partial y = -j\omega\mu H_x$------------------(12)

$\gamma E_x+\frac{\partial Ez}{\partial x} = j\omega\mu H_y$------------------------(13)

$\partial E_y/\partial x$-$\partial Ex/\partial y$ = $-j\omega\epsilon H_z$---------------(14)

From eqn(13)

$H_y=$ $[\ \gamma E_x+\frac{\partial Ez}{\partial x}]/\ j\omega\mu$ ------------(15)

By substituting eqn(15) in eqn(9) we get

$\gamma^2/j\omega\mu E_x+\gamma/j\omega\epsilon\frac{\partial Ez}{\partial x}+\partial H_z/\partial y= j\omega\epsilon E_x$

since $\gamma^2+\omega^2\mu\epsilon= h^2$

by dividing the above equation with h^2 we get

$E_x= -\gamma/h^2\frac{\partial Ez}{\partial x}- j\omega\mu/h^2\ \partial H_z/\partial y$----------(15)

Similarly

$E_y = -\gamma/h^2\frac{\partial Ez}{\partial x}+j\omega\epsilon/h^2\ \partial E_z/\partial y$----------(16)

And

$H_x= -\gamma/h^2\frac{\partial Hz}{\partial x}+ j\omega\mu/h^2\ \partial E_z/\partial y$----------(17)

$H_y= -\gamma/h^2\frac{\partial Hz}{\partial y}- j\omega\mu/h^2\ \partial E_z/\partial x$----------(18)

These equations give a general relationship for field components with in a waveguide.

Propagation of TEM Waves:

For TEM wave

E_z=0 and H_z=0

Substituting these values in equns (15) to (18) all the field components along x and y directions E_x,E_y,H_x,H_y vanish and have a TEM wave cannot exist inside a waveguide.

Modes

The electromagnetic wave inside a waveguide can have an infinite number of patterns which are called modes.

The electric field cannot have a component parallel to the surface i.e. the electric field must always be perpendicular to the surface at the conductor.

The magnetic field on the other hand always parallel to the surface of the conductor and cannot have a component perpendicular to it at the surface.

TE Mode Analysis

The TE_{mn} modes in a rectangular waveguide are characterized by $E_Z=0$. The z component of the magnetic field, H_Z must exist in order to have energy transmission in the guide.

The wave equation for TE wave is given by

$\nabla 2Hz = -\omega 2\mu\epsilon Hz$--------(1)

i.e. $\frac{\partial 2Hz}{\partial x2} + \frac{\partial 2Hz}{\partial y2} + \frac{\partial 2Hz}{\partial z2} = -\omega 2\mu\epsilon Hz$

$\frac{\partial 2Hz}{\partial x2} + \frac{\partial 2Hz}{\partial y2}+\gamma 2H_Z+\omega^2\mu\epsilon H_Z=0$

$\frac{\partial 2Hz}{\partial x2} + \frac{\partial 2Hz}{\partial y2}+(\gamma 2+\omega^2\mu\epsilon)H_Z=0$ $\quad\quad\quad$ $\gamma 2+\omega^2\mu\epsilon=h^2$

$\frac{\partial 2Hz}{\partial x2} + \frac{\partial 2Hz}{\partial y2}+h^2\ H_Z=0$--------(2)

This is a partial differential equation whose solution can be assumed.

Assume a solution

$H_Z=XY$

Where X=pure function of x only

Y= pure function of y only

From equation 2

$\frac{\partial 2[XY]}{\partial x2} + \frac{\partial 2[XY]}{\partial y2}+h^2\ XY=0$

$Y\frac{\partial 2X}{\partial x2} + X\partial 2Y/\partial y2+h2\ XY=0$

Dividing above equation with XY on both sides

$1/X\frac{\partial 2X}{\partial x2} + 1/Y\partial 2Y/\partial y2+h^2 =0$--------(3)

Here $1/X\frac{\partial 2X}{\partial x2}$ is purely a function of x and $1/Y\partial 2Y/\partial y2$ is purely a function of y

Let $1/X\frac{\partial 2X}{\partial x2} = -B^2$ & $1/Y\partial 2Y/\partial y2 = -A^2$

i.e. from equation (3)

$-B^2-A^2+h^2=0$

i.e. $h^2=A^2+B^2$-------(4)

$X=c_1\cos Bx+c_2\sin Bx$

$Y=c_3\cos Ay+c_4\sin Ay$

i.e. the complete solution for $H_z=XY$ is

H_Z= ($c_1$cosBx+$c_2$sinBx)($c_3$cosAy+$c_4$sinAy)----(5)

Where c_1,c_2,c_3 and c_4 are constants which can be evaluated by applying boundary conditions.

Boundary Conditions

Since we consider a TE wave propagating along z direction. So E_Z=0 but we have components along x and y direction.

E_X=0 waves along bottom and top walls of the waveguide

E_y=0 waves along left and right walls of the waveguide

1st Boundary condition:

E_X=0 at y=0 ∀ x→ 0 to a(bottom wall)

2nd Boundary condition

E_X=0 at y=b ∀ x→ 0 to a (top wall)

3rd Boundary condition

E_y=0 at x=0 ∀ y→ 0 to b (left side wall)

4th Boundary condition

E_y=0 at x=a ∀ y→ 0 to b (right side wall)

i) Substituting 1st Boundary condition in eqn(5)

Since we have

$E_X= -\gamma/h^2 \partial E_Z/\partial x - j\omega\mu/h^2 \partial H_Z/\partial y$-----(6)

Since E_Z=0 → $E_X= -j\omega\mu/h^2 \partial[(c1\cos Bx + c2\sin Bx)(c3\cos Ay + c4\sin Ay)/\partial y]$

$E_X= -j\omega\mu/h^2 \partial[(c1\cos Bx + c2\sin Bx)(-A\, c3\sin Ay + Ac4\cos Ay)/\partial y]$

From the first boundary condition we get

$0= -j\omega\mu/h^2 \partial[(c1\cos Bx + c2\sin Bx) \neq 0, A\neq 0$

c4=0

Substituting the value of c4 in eqn (5), the solution reduces to

H_Z= (c1cosBx+c2sinBx)(c3cosAy)--------(7)

ii) from third boundary condition

E_y=0 at x=0 ∀ y→ 0 to b

Since we have

$E_y= -\gamma/h^2 \partial E_z/\partial y + j\omega\mu/h^2 \partial H_z/\partial x$------------(8)

Since E_z=0 and substituting the value of H_z in eqn(7), we get

$E_y= j\omega\mu/h^2 \partial[(c1\cos Bx + c2\sin Bx)(c3\cos Ay)]/\partial x$

$E_y= j\omega\mu/h^2 [(-Bc1\sin Bx + Bc2\sin Bx)(c3\cos Ay)]$

From third condition,

$0=j\omega\mu/h^2(0+Bc2)c3\cos Ay$

Since $\cos Ay\neq0, B\neq 0$, c3#0

$c2=0$

from eq (7)

$H_z=c1c3\cos Bx\cos Ay$---------(9)

iii) 2nd Boundary condition

since we have

$E_x= -\gamma/h^2\partial E_Z/\partial x - j\omega\mu/h^2\partial H_Z/\partial y$

$= -j\omega\mu/h^2\partial/\partial y$[c1c3cosBxcosAy] [$E_Z=0$]

$E_x = j\omega\mu/h^2$ c1c3cosBxsinAy

From the second boundary condition,

$E_x=0$ at y=b ∀ x→ 0 to a

$0= j\omega\mu/h^2$ c1c3cosBxsinAb

cosBx#0,c1c3#0

sinAb=0 or Ab=nπ where n=0,1,2----

A=nπ/b------(10)

iv) 4th Boundary condition

since

$E_y= -\gamma/h^2\partial E_z/\partial y + j\omega\mu/h^2\partial H_z/\partial x$

$E_y= -j\omega\mu/h^2\partial/\partial x$[c1c3cosBxcosAy]

$E_y= -j\omega\mu/h^2$c1c3sinBx.BcosAy

From the 4th Boundary condition

$E_y=0$ at x=a ∀ y→ 0 to b

0= -jωμ/h2Bc1c3sinBx.cosAy ∀ y→ 0 to b

cosAy#0,c1c3#0

sinBa=0

B=mπ/a-------(11)

From eq(9)

H_z= c1c3cos(mπ/a)xcos(nπ/b)y

Let c1c3=c

H_Z=ccos(mπ/a)xcos(nπ/b)y$e^{(j\omega t-\gamma z)}$---------(12)

Field Components

E_x= $-\gamma/h^2 \partial E_Z/\partial x$-$j\omega\mu/h^2 \partial H_{Z/}\partial y$

Since E_z=0 for TE wave

E_x=$j\omega\mu/h^2$c(nπ/b)cos(mπ/a)xsin(nπ/b)y$e^{(j\omega t-\gamma z}$--------(13)

E_y= $-\gamma/h^2 \partial E_z/\partial y$+$j\omega\mu/h^2 \partial H_z/\partial x$

Since E_z=0 for TE wave

E_y= $j\omega\mu/h^2 \partial H_z/\partial x$

E_y= $-j\omega\mu/h^2$c[mπ/a]sin(mπ/a)xcos(nπ/b)y$e^{(j\omega t-\gamma z)}$---------(14)

Similarly

H_x= $-\gamma/h^2 \partial H_Z/\partial x$-$j\omega\epsilon/h^2 \partial E_{Z/}\partial y$

H_x= γ/h^2c(mπ/a)sin(mπ/a)xcos(nπ/b)y$e^{(j\omega t-\gamma z)}$----------------(15)

H_y= $-\gamma/h^2 \partial H_z/\partial y$-$j\omega\epsilon/h^2 \partial E_z/\partial x$

H_y= $-\gamma/h^2$c$(n\pi/b)^2$cos(mπ/a)x.sin(nπ/b)y$e^{j\omega t-\gamma z}$----------------(16)

TM Mode Analysis

For TM wave H_z=0 E_z#0

$\partial^2 E_z/\partial x^2+\partial^2 E_z/\partial y^2+h^2 E_z=0$--------(1)

This is a partial differential equation which can be solved to get the different field components E_x,E_y,H_x and H_y by variable separable method.

Let us assume a solution

E_z=XY------------(2)

Using these two equations from eqn(1) we get

Y$\frac{\partial 2X}{\partial x2} + X\partial 2Y/\partial y2$+h2 XY=0-----(3)

Dividing above equation with XY on both sides

1/X$\frac{\partial 2X}{\partial x2} + 1/Y\partial 2Y/\partial y2$+$h^2$ =0--------(4)

Here 1/X$\frac{\partial 2X}{\partial x2}$ is purely a function of x and $1/Y\partial 2Y/\partial y2$ is purely a function of y

Let $1/X\frac{\partial 2X}{\partial x2} = -B^2$-----------(5)

$1/Y\partial 2Y/\partial y2 = -A^2$-----------(6)

i.e. from equation (4),(5) and(6)

$-B^2-A^2+h^2=0$

i.e. $h^2=A^2+B^2$-------(7)

the solution of eqn(5) and(6) are

$X=c_1\cos Bx+c_2\sin Bx$

$Y=c_3\cos Ay+c_4\sin Ay$

Where c_1,c_2,c_3 and c_4 are constants which can be evaluated by applying boundary conditions

From eqn(1)

$E_Z= XY$

$E_Z= (c_1\cos Bx+c_2\sin Bx)(c_3\cos Ay+c_4\sin Ay)$----(10)

Boundary Conditions

Since we consider a TE wave propagating along z direction. So $E_Z=0$ but we have components along x and y direction.

$E_X=0$ waves along bottom and top walls of the waveguide

$E_y=0$ waves along left and right walls of the waveguide

1st Boundary condition:

$E_X=0$ at $y=0$ $\forall$ $x\rightarrow$ 0 to a(bottom wall)

2nd Boundary condition

$E_X=0$ at $y=b$ $\forall$ $x\rightarrow$ 0 to a (top wall)

3rd Boundary condition

$E_y=0$ at $x=0$ $\forall$ $y\rightarrow$ 0 to b (left side wall)

4th Boundary condition

$E_y=0$ at $x=a$ $\forall$ $y\rightarrow$ 0 to b (right side wall)

i) Substituting 1st Boundary condition in eqn(10)

Since we have

$0=E_Z= [c_1\cos Bx+c_2\sin Bx][c_3\cos A0+c_4\sin A0]$

[c1cosBx+c2sinBx]c3=0

c1cosBx+c2sinBx#0

c3=0

i.e. E_z=[c1cosBx+c2sinBx]c4sinAy--------(11)

ii) Substituting 2nd Boundary condition in eqn(11), we get

E_z=c2c4sinBxsinAy------(12)

iii) Substituting 3rd Boundary condition in eqn(12), we get

sinAb=0

A=nπ/b--------(13)

iv) Substituting 4th Boundary condition in eqn(12), we get

sinBa=0

B=mπ/a-------(14)

From (12),(13),(14)

$E_z = c\sin(m\pi/a)x\sin(n\pi/b)y e^{j(\omega t-\gamma z)}$----------(15)

$E_x = -\gamma/h^2 \partial E_z/\partial x$

$E_x = -\gamma/h^2 c(m\pi/a)\cos(m\pi/a)x\sin(n\pi/b)y e^{j\omega t-\gamma z}$------(16)

$E_y = -\gamma/h^2 c(n\pi/b)\sin(m\pi/a)x\cos(n\pi/b)y e^{j\omega t-\gamma z}$--------(17)

$H_x = j\omega\epsilon/h^2 c(n\pi/b)\sin(m\pi/a)x\cos(n\pi/b)y e^{j\omega t-\gamma z}$--------(18)

$H_y = j\omega\epsilon/h^2 c[m\pi/a]\cos(m\pi/a)x\sin(n\pi/b)y e^{j\omega t-\gamma z}$--------(19)

Cut-off Frequency of a Waveguide

Since we have

$\gamma^2+\omega^2\mu\epsilon=h^2=A^2+B^2$

A=nπ/b,B=mπ/a

$\gamma^2=(m\pi/a)^2+(n\pi/b)^2-\omega^2\mu\epsilon$

$\gamma=\sqrt{(\frac{m\pi}{a})^2+(n\pi/b)^2-\omega^2\mu\epsilon}=\alpha+j\beta$

At lower frequencies

$\gamma > 0$

$\sqrt{(\frac{m\pi}{a})^2+(n\pi/b)^2-\omega^2\mu\epsilon} > 0$

γ then becomes real and positive and equal to the attenuation constant α i.e. the wave is completely attenuated and there is no phase change. Hence the wave cannot propagate.

However at higher frequencies, $\gamma<0$

$\sqrt{(\frac{m\pi}{a})^2+(n\pi/b)^2-\omega^2\mu\epsilon}<0$

γ becomes imaginary there will be phase change β and hence the wave propagates.

At the transition γ becomes zero and the propagation starts. The frequency at which γ just becomes zero is defined as the cut-off frequency f_c

At $f=f_c, \gamma=0$

$0=(m\pi/a)^2+(n\pi/b)^2-\omega_c^2\mu\epsilon$ or

$f_c=1/2\pi\sqrt{\mu\epsilon}[(m\pi/a)^2+(n\pi/b)^2]^{1/2}$

$f_c=c/2[(m\pi/a)^2+(n\pi/b)^2]^{1/2}$

The cut-off wavelength(λ_c) is

$\lambda_c=c/f_c=c/c/2[(m\pi/a)^2+(n\pi/b)^2]^{1/2}$

$\lambda_{cm,n}=2ab/[m^2b^2+n^2a^2]^{1/2}$

All wavelengths greater than λ_c are attenuated and these less than λ_c are allowed to propagate inside the waveguide.

Guided Wavelength (λ_g)

It is defined as the distance travelled by the wave in order to undergo a phase shift of 2π radians.

It is related to phase constant by the relation

$\lambda_g=2\pi/\beta$

the wavelength in the waveguide is different from the wavelength in free space. Guide wavelength is related to free space wavelength λ_0 and cut-off wavelength λ_c by

$1/\lambda_g^2=1/\lambda_0^2-1/\lambda_c^2$

The above equation is true for any mode in a waveguide of any cross section

Phase Velocity(v_p)

Wave propagates in the waveguide when guide wavelength λ_g is grater than the free space wavelength λ_0.

In a waveguide, $v_p=\lambda_g f$ where v_p is the phase velocity. But the speed of light is equal to product of λ_0 and f.This v_p is greater then the speed of light since $\lambda_g>\lambda_0$.

The wavelength in the guide is the length of the cycle and v_p represents the velocity of the phase.

It is defined as the rate at which the wave changes its phase interms of the guide wavelength.

$V_p=\omega/\beta$

$V_p=c/[1-(\lambda_0/\lambda_c)^2]^{1/2}$

Group Velocity(v_g)

The rate at which the wave propagates through the waveguide and is given by

$V_g=d\omega/d\beta$

Since $\beta=[\mu\epsilon(\omega^2-\omega_c^2)]^{1/2}$

Now differentiating β w.r.t ω we get

$V_g= c[1-(\lambda_0/\lambda_c)^2]^{1/2}$

Consider the product of V_p and V_g

$V_P.V_g=c^2$

Dominant Mode

The mode for which the cut-off wavelength assumes a maximum value.

$\lambda cmn=\frac{2ab}{\sqrt{m2b2+n2a2}}$

Dominant mode in TE

For TE_{01} mode $\lambda_{c01}=2b$

TE_{10} mode $\lambda_{c10}=2a$

Among all λ_{c10} has the maximum value since 'a' is the larger dimensions than 'b'. Hence TE_{10} mode is the dominant mode in rectangular waveguide.

Dominant Mode in TM

Minimum possible mode is TM_{11}. Higher modes than this also exist.

Degenerate Modes

Two or more modes having the same cut-off frequency are called 'Degenerate modes'

For a rectangular waveguide TE_{mn}/TM_{mn} modes for which both m#0,n#0 will always be degenerate modes.

Wavelengths and Impedance Relations[TE &TM WAVES]

Guide Wavelength(λ_g)

It is defined as the distance travelled by the wave in order to under go a phase shift of 2π radians.

$1/\lambda_g^2=1/\lambda_0^2-1/\lambda_c^2$

Wave impedance is defined as the ratio of the strength of electric field in one transverse direction to the strength of the magnetic field along the other transverse direction.

$Z_Z=E_x/H_y$

1) Wave impedance for a TM wave in rectangular waveguide

$Z_Z= -\gamma/h^2\partial E_z/\partial x-j\omega\mu\partial H_z/\partial y/-\gamma/h^2\partial H_z/\partial y-j\omega\epsilon/h^2\partial E_z/\partial x$

For a TM wave $H_z=0$

$Z_{TM}= \gamma/j\omega\epsilon$

$= \beta/\omega\epsilon$

Since we have $\beta=[\omega^2\mu\epsilon-\omega_c^{2\mu\epsilon}]^{1/2}$

$Z_{TM}= \dot{\eta}[1-(\lambda_0/\lambda_c)^2]^{1/2}$

Since 'λ_0 is always less than λ_c for wave propagation $Z_{TM}<\dot{\eta}$

2) Wave impedance of TE waves in rectangular waveguide

$Z_{TE}= \dot{\eta}/[1-(\lambda_0/\lambda_c)^2]^{1/2}$

Therefore $Z_{TE}> \dot{\eta}$

For TEM waves between parallel planes the cut-off frequency is zero and wave impedance for TEM wave is the free space impedance itself

$Z_{TEM}= \dot{\eta}$

Microstrip Line

Microstrip Line is an unsymmetrical stripline that is nothing but a parallel plate transmission line having dielectric substrate, the one face of which is metallised ground and the other face has a thin conducting strip of certain width 'w' and thickness 't' some times a cover plate is used for shielding purposes but it is kept much farther away than the ground plane so as not to affect the microstrip field lines.